GALCIT REPORT NO. ________

GUGGENHEIM AERONAUTICAL LABORATORY

CALIFORNIA INSTITUTE OF TECHNOLOGY

AN INVESTIGATION OF THE STRESSES AND DEFLECTIONS OF SWEPT PLATES

Thesis by

Ralph Stewart Chandler

Thesis
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PASADENA, CALIFORNIA

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Ralph Stewart Chandler

In Partial Fulfillment of the Requirements
for the Degree of
Aeronautical Engineer

California Institute of Technology
Pasadena, California

1949

ACKNOWLEDGEMENT

The author wishes to express appreciation to Mr. Harold De Groff for his constant help in the experimental work and his suggestions concerning the analysis, and to Mr. Harold Lurie and Dr. Y. C. Fung for their help in the analysis.

He also wishes to express his gratitude to Dr. E. E. Sechler and the members of the GALCIT staff for their help in carrying out this investigation.

SUMMARY

The problem in this investigation was to determine the stress and deflection patterns of a thick cantilever plate at various angles of sweepback.

The plate was tested at angles of sweepback of zero, twenty, forty, and sixty degrees under uniform shear load at the tip, uniformly distributed load and torsional loading.

For all angles of sweep and for all types of loading the area of critical stress is near the intersection of the root and trailing edge. Stresses near the leading edge at the root decreased rapidly with increase in angle of sweep for all types of loading. In the outer portion of the plate near the trailing edge the stresses due to the uniform shear and the uniformly distributed load did not vary for angles of sweep up to forty degrees. For the uniform shear and the uniformly distributed loads for all angles of sweep the area in which end effect is pronounced extends from the root to approximately three quarters of a chord length outboard of a line perpendicular to the axis of the plate through the trailing edge root. In the case of uniform shear and uniformly distributed loads the deflections near the edge at seventy-five per cent semi-span decreased with increase in angle of sweep. Deflections near the trailing edge under the same loading conditions increased with increase in angle of sweep for small angles and then decreased at the higher angles of sweep. The maximum deflection due to torsional loading increased with increase in angle of sweep.

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This investigation was carried out at the Guggenheim Aeronautical Laboratory at the California Institute of Technology in conjunction with Commander F. B. Gilkeson, during the academic year 1948 - 1949.

INTRODUCTION

The problem in this investigation was to determine the effect of sweepback upon the deflection and stress pattern of a thick cantilever plate. The plate was tested at angles of sweep of zero, twenty, forty, and sixty degrees under uniform shear, uniformly distributed, and torsional loadings.

This research is one phase of the investigation being carried out at the Guggenheim Aeronautical Laboratory of the California Institute of Technology (GALCIT) to determine the effect of sweep upon the deflection and stress patterns of aircraft wings of high solidity. This work is being carried out both experimentally and theoretically under a contract with the United States Air Force.

Since little experimental data has been published on this phase of structural research, it was necessary to begin the overall investigation of this problem with the study of solid plates having the shape of swept wings and subjected to uniform shear loading, uniformly distributed loading, and torsional loading. By September of 1948 a preliminary investigation on a thin plate had been completed by the GALCIT staff. This work "pointed the way" to the present investigation. This paper will suggest several points to be considered in further experimental investigation of the problem and will furnish data which will be useful as a guide to the correct theoretical solution

of this problem.

This investigation was carried out in the GALCIT structures laboratory under the supervision of Dr. E. E. Sechler. It was done in conjunction with Commander F. B. Gilkeson, U. S. Navy, during the academic year 1948 - 1949.

EQUIPMENT

The test specimen was a 24ST aluminum alloy plate. The plate was one inch thick and the area was maintained constant at 400 square inches at all angles of sweep by cutting triangular pieces parallel to the root from the tip of the plate so as to maintain a constant length of forty inches and a constant width of ten inches. A square grid was scribed on the bottom of the plate at two and one-half inch intervals along and across the plate to facilitate the measuring of deflections. The dimensions of the specimen in the four configurations of zero, twenty, forty, and sixty degrees of sweep are shown in Figures 4 through 15.

Standard SR-4 strain rosettes manufactured by the Baldwin-Southwark Company were attached to the specimen at the points indicated in Figures 4 through 15. These rosettes were connected to a wheatstone bridge circuit from which strain readings in millivolts were taken. These readings were converted into principal stresses. The electrical setup is shown in Figure 3.

The plate was supported by a heavy framework of I beams and steel plates. This support is shown in Figures 1 and 2. The framework was bolted to a concrete floor. This method of supporting the plate gave a reasonable degree of fixity. As complete fixity was not possible, a survey was made to determine the amount of "sag" of the support.

The test plate was placed between two steel plates at the top of the support. In order to obtain a uniform pressure and a maximum fixity, specially cut spacers were inserted between the test plate and the supporting steel plates.

A dial deflection gage manufactured by the B. C. Ames Company of Waltham, Massachusetts was used to measure deflection. This gage was calibrated to one thousandth of an inch. A large smooth table was used to support this gage.

PROCEDURE

The plate was tested under three types of loadings at angles of sweepback of zero, twenty, forty, and sixty degrees. These loadings are referred to as uniform shear, uniformly distributed, and torsion loads. The uniform shear load was applied at the tip by means of a whiffle tree arrangement. This permitted the uniform shear load to be applied uniformly across the tip of the plate. Shot bags were placed in a large pan supported by the whiffle tree to give the desired load. Uniform shear loads of two hundred, four hundred, and six hundred pounds were applied for each angle of sweep. (See Figure 1.)

Uniformly distributed loads of one, two, and three pounds per square inch were applied at each angle of sweep. These loads were applied by placing shot bags uniformly over the surface of the test plate. A mat of sponge rubber was placed over the plate to protect the strain rosettes from the shot bags. Uniformly distributed loads of one, two, and three pounds per square inch were used in order to make the moment at the root caused by these loads equal the moments at the root caused by the two hundred, four hundred, and six hundred pounds uniform shear loads.

Torsion loads of fifteen thousand, thirty thousand, and forty-five thousand inch pounds were applied at each angle of sweep. These torsion loads were applied at the tip by means of an I beam bolted to

the tip. Pans were attached to the ends of the I beams by flexible steel cables. The cable on the side of leading edge lead vertically downward. The cable on the side of the trailing edge was lead vertically upward to a pulley and then downward. Shot bags were loaded in the pans to give the desired load. (See Figure 2). The bolt holes in the tip used for fastening the I beam to the plate were bored parallel to the axis of the plate. This resulted in a torsional load vector perpendicular to the tip as shown in Figures 2, 5, 8, 11, and 14.

Deflections for all types of loads were obtained by measuring the change in distance between a smooth table and the plate when the various loads were applied. Zeros were obtained before and after loading and it was found that at least three loading cycles were needed in order to stabilize these zero readings. Deflection readings were taken at five inch intervals span wise and at the zero, twenty-five, fifty, seventy-five, and one hundred per cent chord points. These deflections are plotted in Figures 16 through 27. These curves are not corrected for the sag of the support. For the concentrated loads and the uniform loads, deflections in the direction of loading are plotted as positive deflections. For torsional loads, up deflections are plotted as positive deflections and down deflections are plotted as negative deflections.

The orientation and magnitude of principal stresses at the

various strain rosette locations were plotted as shown in Figures 4 to 15.

In order to determine the sag of the support, a lightweight I beam was fastened to the top of the support and its deflection was measured when the plate was loaded. The deflection of the top support plate was measured by mounting an arch on the ends of this plate and measuring the change in the distance between this arch and the support plate upon loading the test plate. The sag of the bottom support plate was measured in a manner similar to the method used to measure the deflection of the test plate. The sag due to torsion loads, i.e., in the plane of the support, was found to be negligible. The corrections due to the sag of the support for the uniform shear and uniformly distributed loads are shown in Figure 36. All deflection plots subsequent to Figure 27 have been corrected for the sag of the support.

Cross plots were made to show the variation in deflection with increased angle of sweep for points on the fifty and seventy-five per cent semi-span lines. Figure 34 shows this variation for uniform shear and uniformly distributed loadings, and Figure 35 for torsion loading.

Cross plots to show this variation in stresses near the trailing edge for the various sweep angles were made in Figures 28 to 30. Similar plots were made for the stresses near the leading edge as shown in Figures 31(a) to 33(b). Data for these plots are listed in Tables I and II.

Figures 31(b) and 33(b) are tracings of Figures 31(a) and 33(a) respectively with additional curves representing the strains as calculated by the standard engineering formula for a simple cantilever beam. For these curves the simple beams were considered to be fixed at a line perpendicular to the axis through the trailing edge root. In these calculations for the uniform shear load, the total shear was assumed concentrated at the tip at the fifty per cent chord point.

Since it was found that both stresses and deflections varied linearly with increase in load for all types of loading and for all angles of sweep, all tables and graphs in this report are for the maximum load of each of the various loading conditions.

RESULTS AND DISCUSSION

I. Deflections

The deflections of the test plate for all types of loading and for all angles of sweep are plotted in Figures 16 through 27. These curves have not been corrected for the sag of the support. Corrections for sag of the support are plotted in Figure 36. Curves showing the variation of deflection at fifty and seventy-five per cent semi-span with angle of sweep are plotted in Figures 34 and 35. These curves were corrected for the sag of the support.

For the uniform shear and uniformly distributed loads the deflection of the trailing edge at seventy-five per cent semi-span increased with an increase in angle of sweep up to an angle of sweep between twenty and twenty-five degrees and then decreased. The deflection of the leading edge at seventy-five per cent semi-span decreased with angle of sweep. Both of these effects are due to the fact that as the angle of sweep is increased the bending moment is reduced and an increasing twisting moment is introduced.

In the case of torsional loading the deflection of the trailing edge at seventy-five per cent semi-span increased with increase in angle of sweep. The deflection of the leading edge for the same point spanwise was in the direction of torque for small angles of sweep and opposite to the direction of torque for large angles of sweep. The maximum deflection in this direction was reached at an angle of sweep

of approximately forty-five degrees. This effect is due to the fact that the manner of applying the torque introduced a bending component which caused the plate to bend upward more and more as the angle of sweep increased.

All deflections increased linearly with increase in load for all loading conditions and for all angles of sweep.

At zero angle of sweep the experimental deflections due to the uniform shear load were found to be slightly greater than those computed by the engineering formula for prismatical beams. (See Table 6).

II. Stresses

The magnitude and direction of the principal stresses are plotted in Figures 4 through 15. Variation of the stresses with angle of sweep along the ninety per cent chord line is plotted in Figures 28 through 30. Similar curves along the ten per cent chord line are plotted in Figures 31 through 33.

The stresses near the intersection of the root and the trailing edge were found to increase rapidly for angles of sweep greater than zero for all types of loading. Just the opposite effect was noted for the area near the intersection of the leading edge and the root where the stresses decreased rapidly with increase in angle of sweep for all types of loading. In fact at an angle of sweep of sixty degrees the stresses in this area are practically negligible. For all angles of

sweep and for all types of loading the area of critical stress was near the intersection of the trailing edge and the root.

The stresses along the ninety per cent chord line from fifteen to one hundred per cent semi-span due to the uniform shear load were found to be linear for all angles of sweep. In this portion of the plate the stresses did not change with angle of sweep up to an angle of sweep of forty degrees, then they dropped off slightly for the sixty degree angle of sweep. This second fact was found to hold true for the stresses due to both uniform shear and uniformly distributed loads.

Stresses in the portion of the plate mentioned in the previous paragraph were found to be at least ten per cent less than the stresses as calculated by the engineering formula for angles of sweep up to forty degrees for both uniform shear and uniformly distributed loads.

Compressive stresses along the ninety per cent chord line due to torsional loading increased with increase in angle of sweep and the tensile stresses decreased. This is due to the fact that the bending stresses become larger with increase in angle of sweep due to the manner in which the torque was applied.

Along the ten per cent chord line the point of maximum stress due to all types of loading moved outward with an increase in angle of sweep. The distance from the root along this chord line to the point of maximum stress appears to increase linearly with increase in angle of sweep for both the uniform shear and uniformly distributed loads.

However, it would be necessary to test the plate at additional angles of sweep to establish this fact conclusively. (See Table V).

In calculating the stresses near the leading edge by means of the standard engineering formula for a cantilever beam, the portion of the plate outboard of a line drawn through the trailing edge root and perpendicular to the swept axis of the plate was assumed to act like a simple cantilever beam. The uniform shear load was assumed to be concentrated at the tip and fifty per cent chord point. These assumptions resulted in a different engineering formula curve for each angle of sweep. Outside the area of end effect the experimental results agree very well with the theoretical results for all angles of sweep in the case of the uniformly distributed load. For the uniform shear load outside the area of end effect the engineering formula gives conservative results for zero angle of sweep, agrees very well for the twenty and forty degree angles of sweep and is non-conservative for the sixty degree angle of sweep.

For the uniform shear and the uniformly distributed loads, for all angles of sweep, the experimental stress curve departs from the theoretical curves at a distance of approximately three quarters of a chord length from a line which is perpendicular to the axis of the plate through the trailing edge root, i.e., the assumed root of the cantilever beam used in the engineering formula calculations. This leads to the conclusion that "end effect" extends out this distance.

Stresses varied linearly with increase in load for all angles of sweep and for all types of loading.

III. Accuracy

In order to estimate the accuracy of the results obtained a survey was made of the stresses in the outer portion of the test plate when under maximum torsional loading and at zero angle of sweep. Under these conditions the stresses throughout this portion of the plate should have been equal. The average of all these stresses was obtained and then the maximum and average errors were computed by comparing this average stress with the actual stresses. The maximum error was equal to plus or minus 6.28 per cent and the average error was plus or minus 2.72 per cent. The accuracy of these results was checked by multiplying the stresses due to one-third maximum load by three and by multiplying the stresses due to two-thirds maximum load by three halves and comparing these results with the stresses due to maximum load. The maximum and average errors found in this manner were slightly less than those found previously.

The errors in the stress results were due to the inherent error in the strain rosettes, slight variation in the electrical zero, and personnel error.

A comparison of the experimental deflections of the plate at zero angle of sweep with the theoretical deflections as computed by the enginerring beam formula is shown in Table VI. The deflections were also computed using Stevenson's formula as given in Ref. (a). The deflections computed by this formula agreed almost exactly with those computed by the engineering formula. In Stevenson's formula he sets the boundary conditions at only one point, the center of the plate at the root

where he assumes zero deflection and zero slope. Whereas, in this investigation the plate was clamped along the entire root. This fixed end condition leads to more boundary conditions than there are unknown constants in Stevenson's formula. For this reason it is felt that the engineering formula gives as good theoretical results as any known solution for the plate as tested in this investigation. A comparison of experimental results and theoretical results shows a deflection error of approximately three per cent for the maximum deflection. This error is greater at smaller deflections. The error in deflection readings is due to zero reading error and sag of the support in addition to that sag which was measured.

IV. Recommendations

In this experiment it was found that there were too few strain rosettes in the area of critical stress near the root. In the future it is recommended that as many strain rosettes as possible be placed in this area.

It is also felt that more valuable information could be obtained in the same length of time by testing the plate at ten degree increments of sweep angle and at maximum loads only.

CONCLUSIONS

1. For angles of sweep greater than zero and for all types of loading the area of critical stress is near the intersection of the root and trailing edge.

2. Stresses near the trailing edge at the root increased rapidly for angles of sweep greater than zero for all types of loading.

3. Stresses near the leading edge at the root decreased rapidly with increase in angle of sweep for all types of loading.

4. For uniform shear and uniformly distributed loads stresses near the trailing edge in the outer eighty-five per cent of the plate did not vary with angles of sweep up to forty degrees. These stresses become smaller at the sixty degree angle of sweep.

5. Use of the standard engineering formula for stresses in a cantilever beam for the uniform shear and the uniformly distributed loads gives good results in the portion of the plate which is free of end effect.

6. The portion of the plate in which end effect is pronounced extends from the root to a distance of three-quarters of a chord length from a line through the trailing edge root perpendicular to the swept axis of the plate.

7. Near the leading edge the point of maximum stress due to all types of loading moved outward with increase in angle of sweep.

8. In the case of uniform shear and uniformly distributed loads the deflections near the leading edge at seventy-five per cent semi-span decreased with increase in angle of sweep. Deflections near the

trailing edge under the same loading conditions increased with increase in angle of sweep for small angles and then decreased at the higher angles of sweep. The maximum deflection due to torsional loading increased with increase in angle of sweep.

REFERENCE

(a) I. S. Sokolnikoff, "Mathematical Theory of Elasticity", McGraw-Hill Book Company, Inc., 1946 - Page 231.

TABLE 1

Stresses at Ninety Per Cent of Chord

Distance*	Stresses (psi) Uniform Shear Load +	Uniformly Distributed Load +	Torsion Load -	Torsion Load +
		$\beta = 0°$		
1.00	12858	11314	14062	4480
5.00	11486	9303	11340	9624
9.00	9855	7054	10554	11512
13.00	8595	5316	11109	10910
17.00	7460	4124	10974	11544
30.00	3267	791	10927	11658
34.00	2025	236	11541	11068
		$\beta = 20°$		
1.20	17408	15486	21985	5497
5.20	12285	10100	17961	8416
9.20	10324	6882	17125	9022
13.20	8726	5080	17148	8952
17.20	7435	3975	17098	8943
30.30	2921	631	16973	8776
34.30	1490	117	17022	8331
		$\beta = 40°$		
3.17	13359	13425	24515	3948
7.17	10979	8375	21497	4879
11.17	9223	6060	20413	5257
15.17	7644	4555	20119	5137
32.17	2010	270	20459	4882
36.17	764	125	17179	3936
		$\beta = 60°$		
2.67	15178	11702	31323	1284
4.67	11471	8383	26394	1902
8.67	9143	5708	23937	2249
12.67	7422	3635	23404	2183
16.67	5677	2467	23016	2253
24.67	3174	817	22328	2038

*Distance is measured in inches from root along chord line.

TABLE 2

Stresses at Ten Per Cent of Chord

Distance*	Stresses (psi)			
	Uniform Shear Load	Uniformly Distributed Load	Torsion Load	
	+	+	-	+
		$\beta = 0^\circ$		
1.00	13053	12550	3358	14619
5.00	11425	9990	9329	11795
9.00	9423	6820	10881	11082
13.00	8555	5428	10346	11846
17.00	7077	3415	10935	10980
26.00	4568	1325	11550	11439
30.00	3198	762	10619	12018
		$\beta = 20^\circ$		
2.30	10350	10148	6776	9700
4.30	11147	10091	10678	8816
8.30	11265	9569	15118	8755
12.30	9722	6331	15788	8996
16.30	8821	4991	16831	8993
20.30	7166	2241	16819	8928
29.30	4323	1185	17269	9246
33.30	2853	362	17203	9397
		$\beta = 40^\circ$		
1.80	4348	4880	183	7577
3.80	6528	6356	3023	5445
5.80	8417	7831	7816	4236
7.80	9747	8498	12210	4219
9.80	10329	8694	15397	4280
13.80	10331	8524	19265	4817
17.80	8697	5446	19171	5308
21.80	4201	4002	19984	5188
25.80	6130	2662	19599	5366
34.80	3105	647	20100	5278

*Distance is measured in inches from root along chord line.

TABLE 2 (Cont'd)

Stresses at Ten Per Cent of Chord

Distance*	Stresses (psi)			
	Uniform Shear Load	Uniformly Distributed Load	Torsion Load	
	+	+	-	+
		$\beta = 60^{0}$		
2.30	684	274	176	3527
4.30	1652	863	918	3093
6.30	4640	1783	1811	2807
8.30	3153	2758	3029	2020
10.30	4394	3778	5284	1383
12.30	6321	4830	9036	1220
14.30	7859	6120	13546	1372
16.30	8765	6609	17325	1703
18.30	9133	6508	19821	1813
22.30	8324	4968	22564	2117
26.30	7141	3976	21842	2653
30.30	6052	2407	23208	2309

*Distance is measured in inches from root along chord line.

TABLE 3

Deflections at Fifty Per Cent Semi-span

β°	Deflections (inches)				
	Uniform Shear Load				
	L.E.	25%C	50%C	75%C	T.E.
0	.463	.463	.469	.466	.464
20	.416	.445	.475	.507	.531
40	.260	.323	.382	.448	.518
60	.055	.119	.209	.326	.445
	Uniformly Distributed Load				
0	.412	.414	.414	.413	.413
20	.392	.410	.434	.455	.475
40	.253	.306	.358	.403	.456
60	.061	.114	.176	.263	.357
	Torsion Load				
0	-.300	-.150	.020	.180	.335
20	-.060	.100	.270	.445	.630
40	.070	.240	.440	.690	.950
60	.010	.150	.390	.770	1.190

TABLE 4

Deflections at Seventy-five Per Cent Semi-span

Deflections (inches)

$\beta°$	L.E.	25%C	50%C	75%C	T.E.
			Uniform Shear Load		
0	.953	.956	.956	.956	.953
20	.852	.897	.942	.985	1.023
40	.648	.727	.802	.902	1.001
60	.258	.388	.520	.672	.838
			Uniformly Distributed Load		
0	.751	.751	.751	.751	.751
20	.739	.761	.787	.810	.837
40	.552	.607	.670	.727	.792
60	.220	.316	.398	.501	.606
			Torsion Load		
0	-.460	-.220	.025	.270	.515
20	.135	.380	.640	.920	1.200
40	.495	.790	1.130	1.500	-----
60	.350	.710	1.180	1.600	-----

Table 5

Variation of Maximum Stress Location with Angle of Sweep
Along Ten Per Cent Chord Line

$\beta°$	Distance from root (inches)
0	0
20	6
40	12
60	18

Table 6

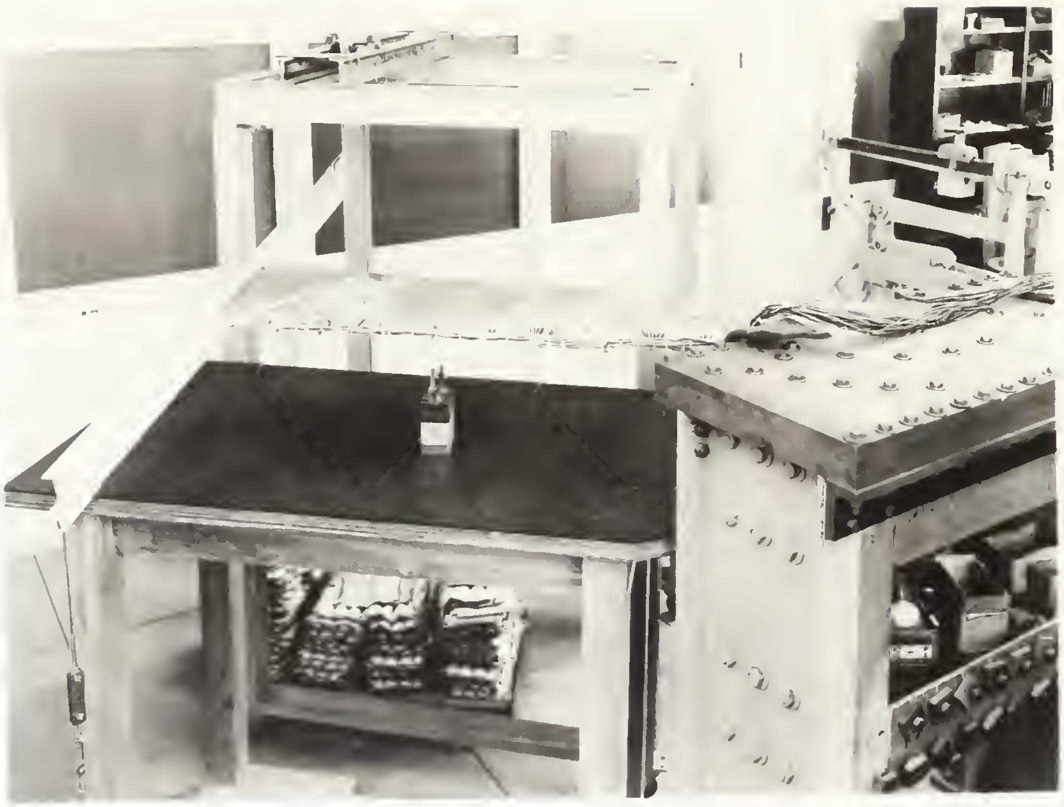
Experimental versus Theoretical Deflection of Cantilever Beam

Distance from Root (inches)	Deflections Theoretical	Experimental	Difference	%
32.5	1.046	1.085	.039	3.7
30.0	.917	.956	.039	4.25
27.5	.792	.830	.038	4.8
22.5	.559	.595	.036	6.43
17.5	.356	.385	.029	8.15
12.5	.190	.218	.028	14.7

FIGURE 1.

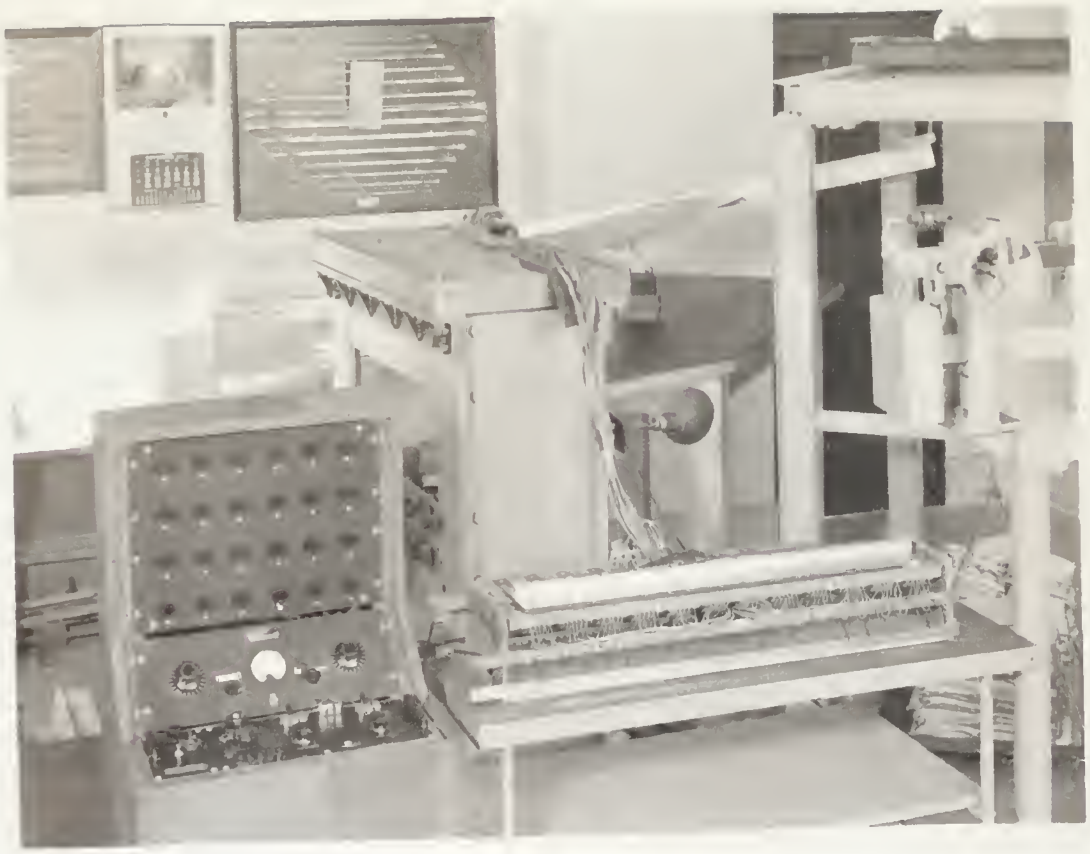
EQUIPMENT UNDER CONCENTRATED LOAD

FIGURE 2.

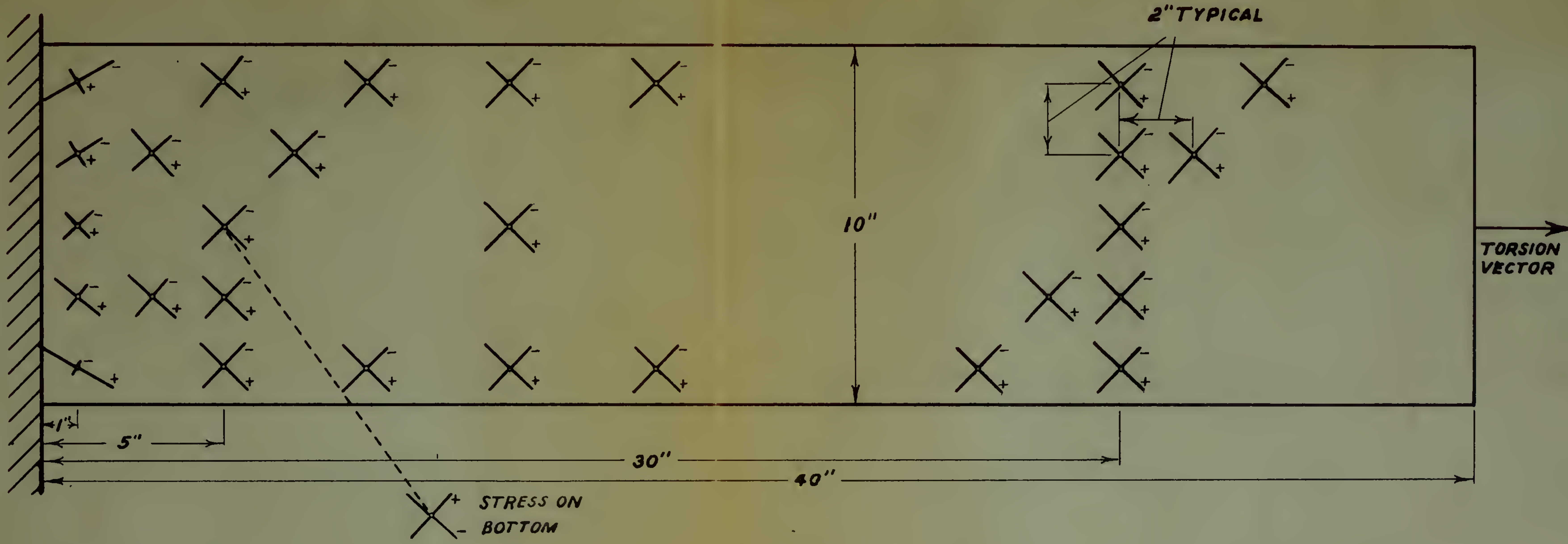


EQUIPMENT UNDER TORSION LOAD

FIGURE 3.



ELECTRICAL EQUIPMENT



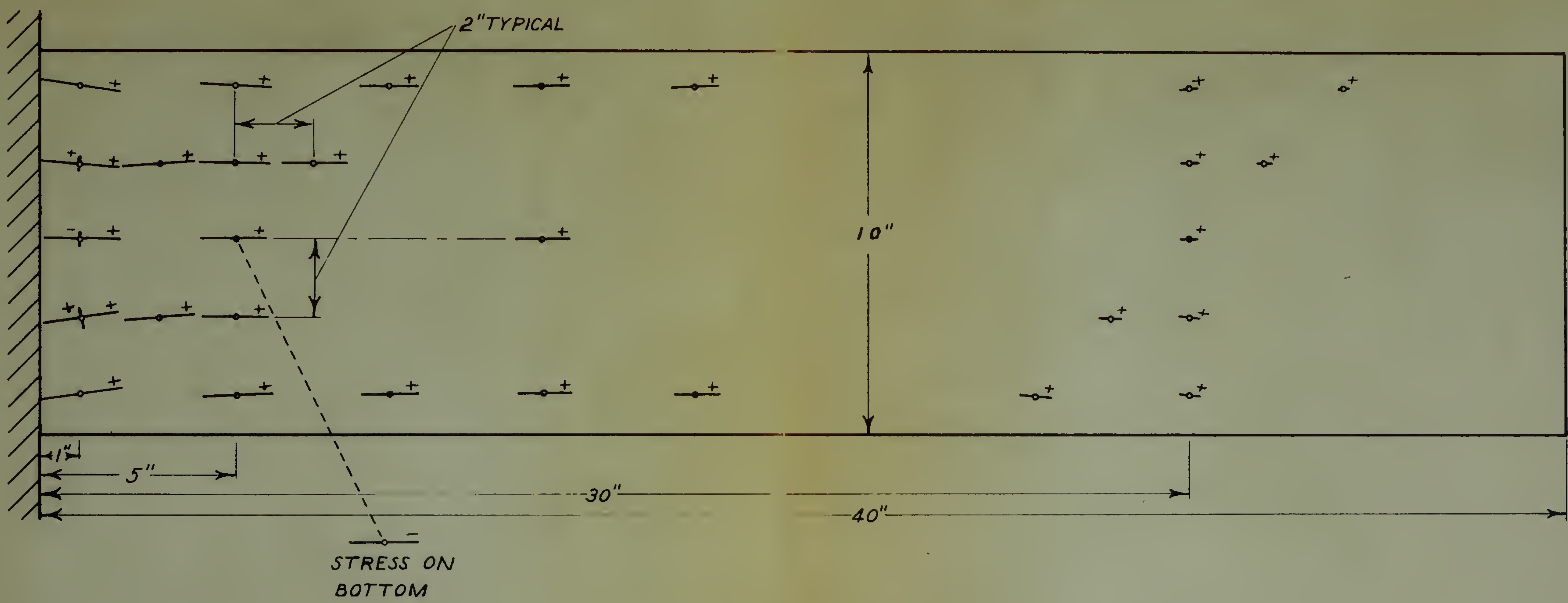
STRESSES PRODUCED BY 45000 IN.-LBS.
TORSION VECTOR PERPENDICULAR TO TIP
$\beta = 0°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. SCALE: 1" = 20X10³ P.S.I.

MODEL #1 TEST #2

Figure 5

			FBG	RSC			TOLERANCES .010 .03 UNLESS OTHERWISE NOTED
			RSC	FBG			2-21-49
MATERIAL	FINISH	HEAT TR.	DRAFTSMAN	CHECKED	APPR.		
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE				
			NAME				DRAW NO. NO.



STRESSES PRODUCED BY 600# LOAD
UNIFORMLY DISTRIBUTED AT TIP
$\beta = 0°$

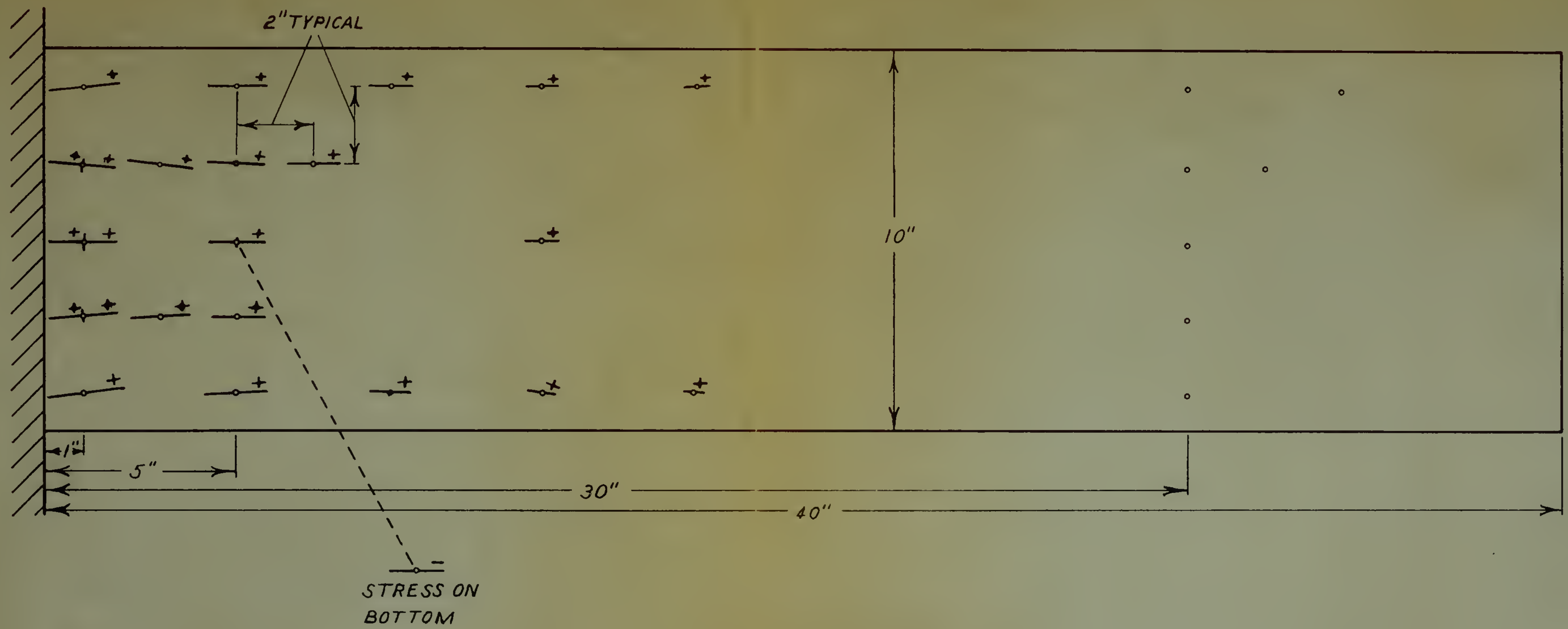
SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20×10^3 P.S.I.

MODEL #1 TEST #1

Figure 4

RSC	FBG	
FBG	RSC	2-21-49

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE



STRESSES PRODUCED BY LOAD OF 3 P.S.I. UNIFORMLY DISTRIBUTED OVER PLATE

$\beta = 0°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGN INDICATES TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20 X 10³ P.S.I.

MODEL #1 TEST # 3

Figure 6

F B G	R S C
R S C	F B G

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

STRESSES PRODUCED BY 600# LOAD
UNIFORMLY DISTRIBUTED AT TIP
$\beta = 20^\circ$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20 X 10³ P.S.I.

MODEL # 2 TEST # 1

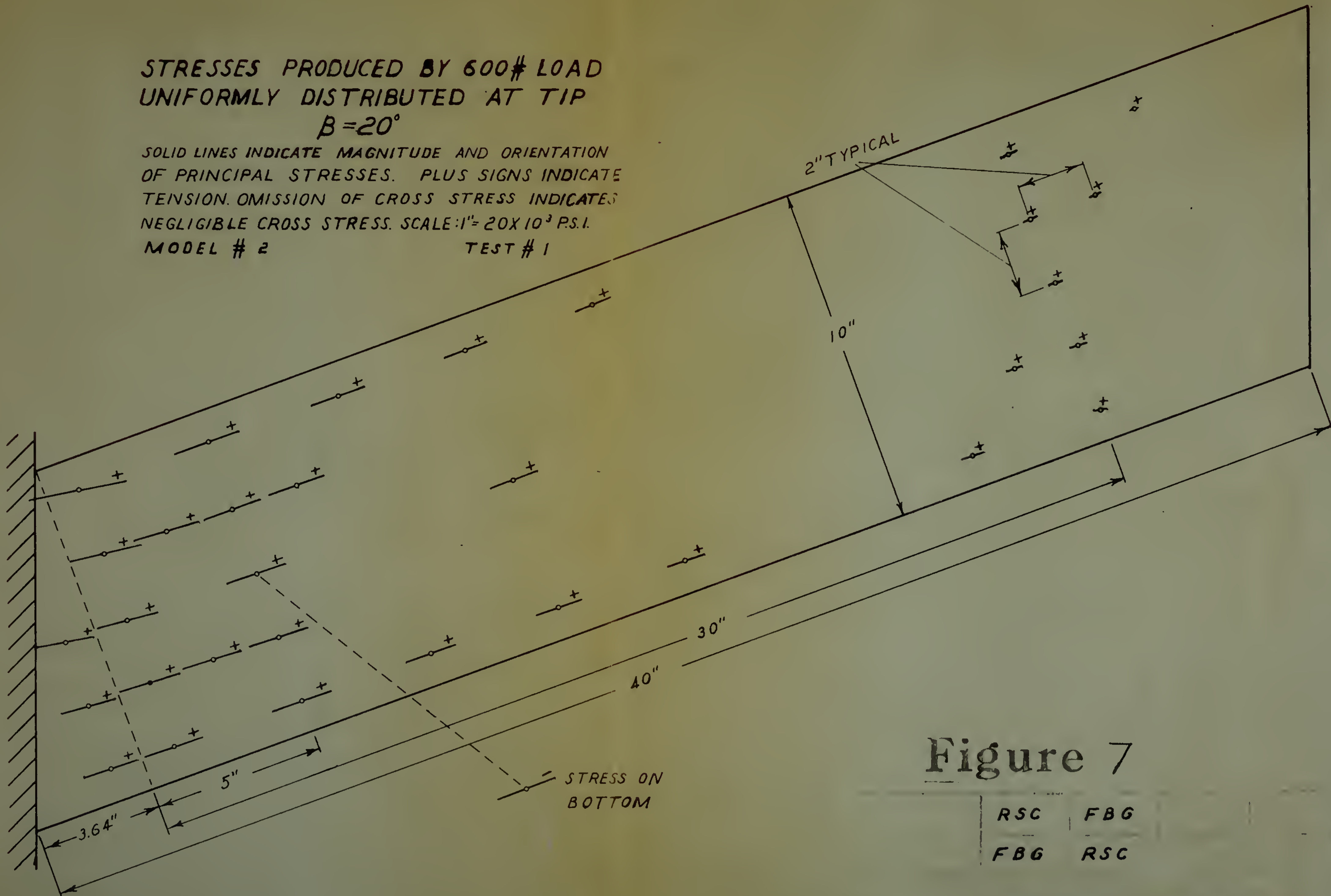
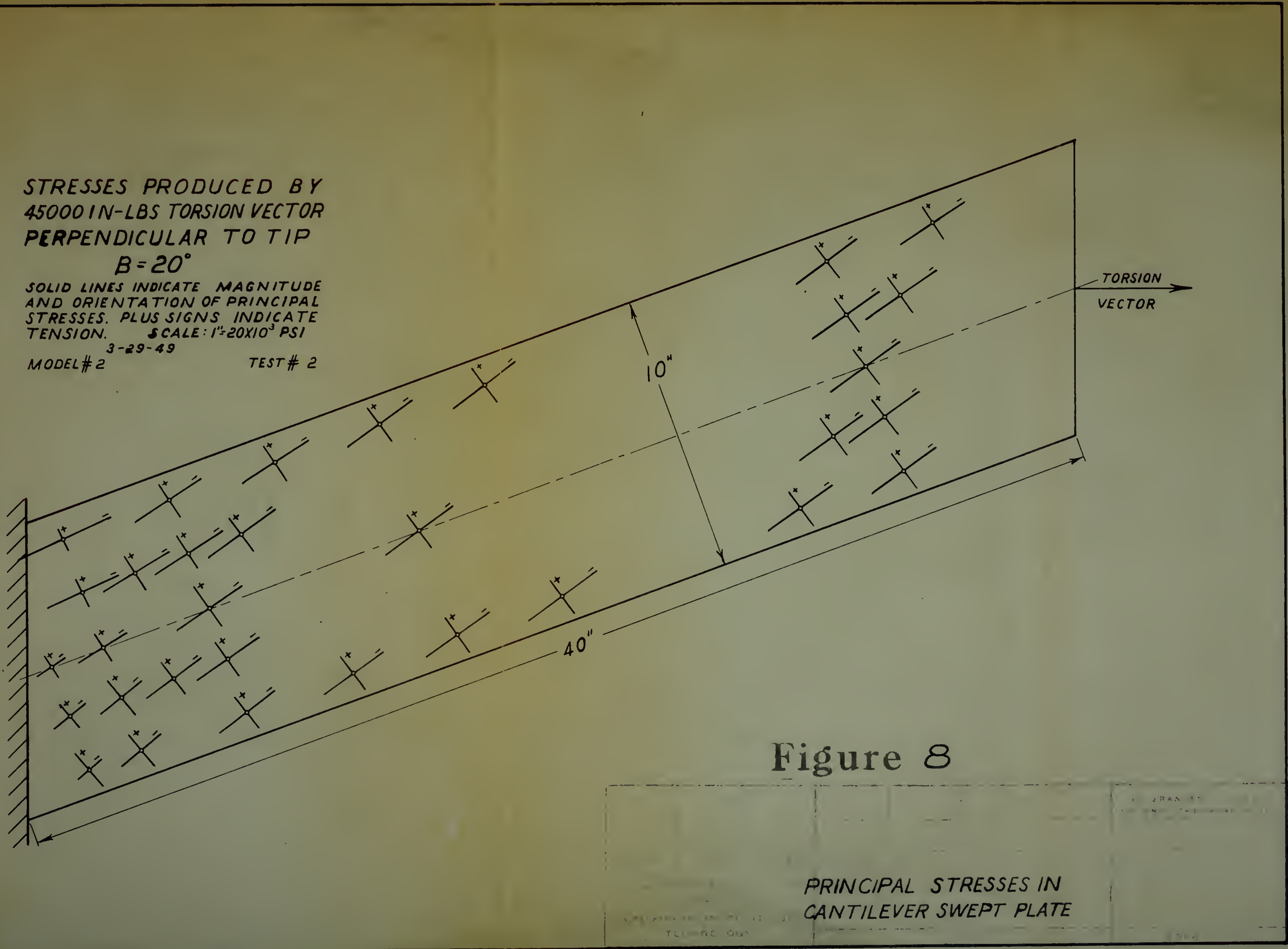


Figure 7

RSC	FBG
FBG	RSC

2-21-49

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE



STRESSES PRODUCED BY
45000 IN-LBS TORSION VECTOR
PERPENDICULAR TO TIP
B = 20°
SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. SCALE: 1" = 20X10³ PSI
3-29-49
MODEL # 2
TEST # 2
TORSION VECTOR
10"
40"
PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

Figure 8

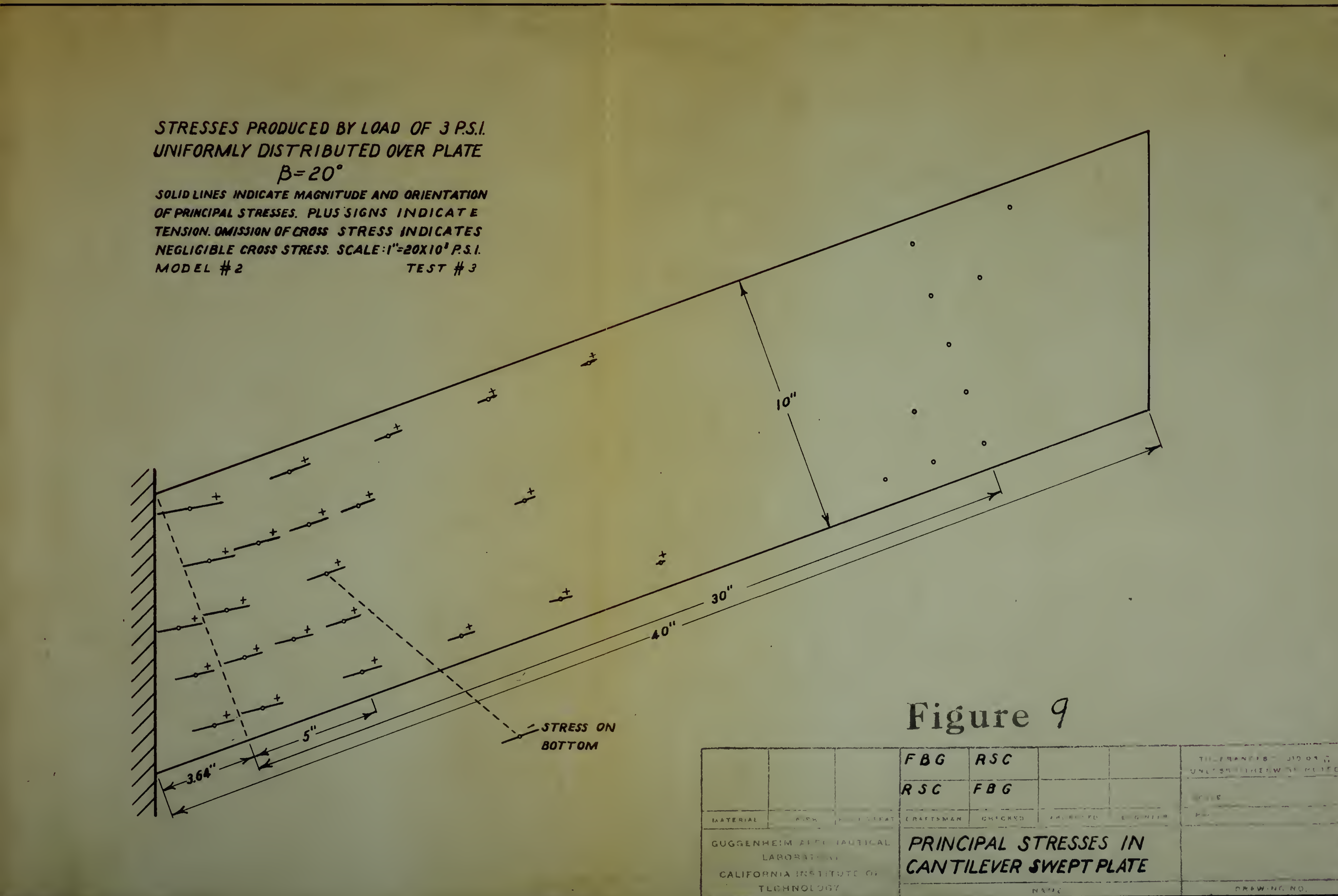

STRESSES PRODUCED BY LOAD OF 3 P.S.I.
UNIFORMLY DISTRIBUTED OVER PLATE
β=20°
SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1"=20X10³ P.S.I.
MODEL #2
TEST #3
10"
30"
40"
5"
3.64"
STRESS ON BOTTOM
FBG
RSC
RSC
FBG
MATERIAL
CRAFTSMAN
CHECKED
GUGGENHEIM AERONAUTICAL LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE
DRAWING NO.

Figure 9

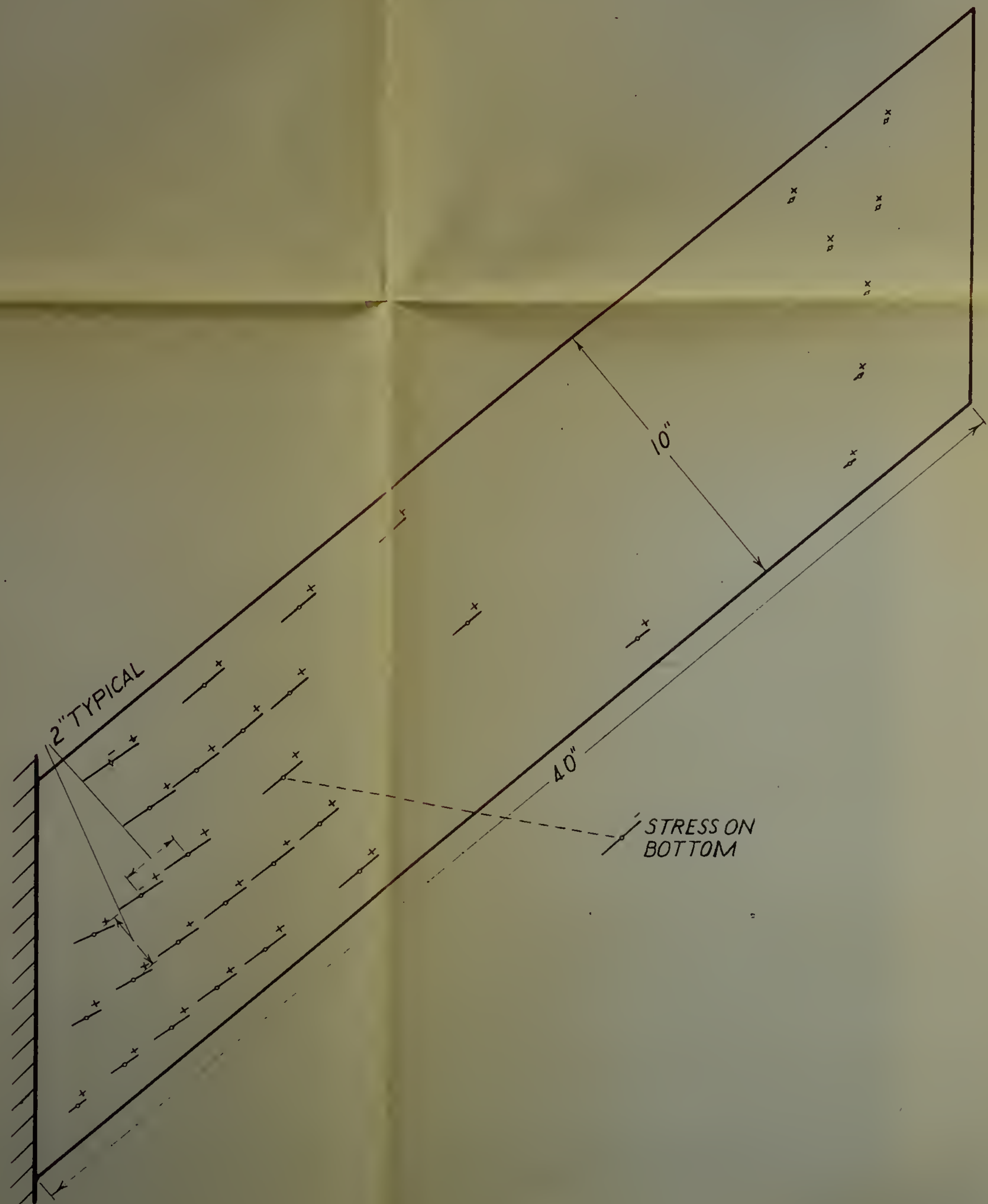


Figure 10

STRESSES PRODUCED BY 600# LOAD
UNIFORMLY DISTRIBUTED AT TIP
$\beta = 40°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20×10^3 PSI
MODEL # 3 TEST # 1

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

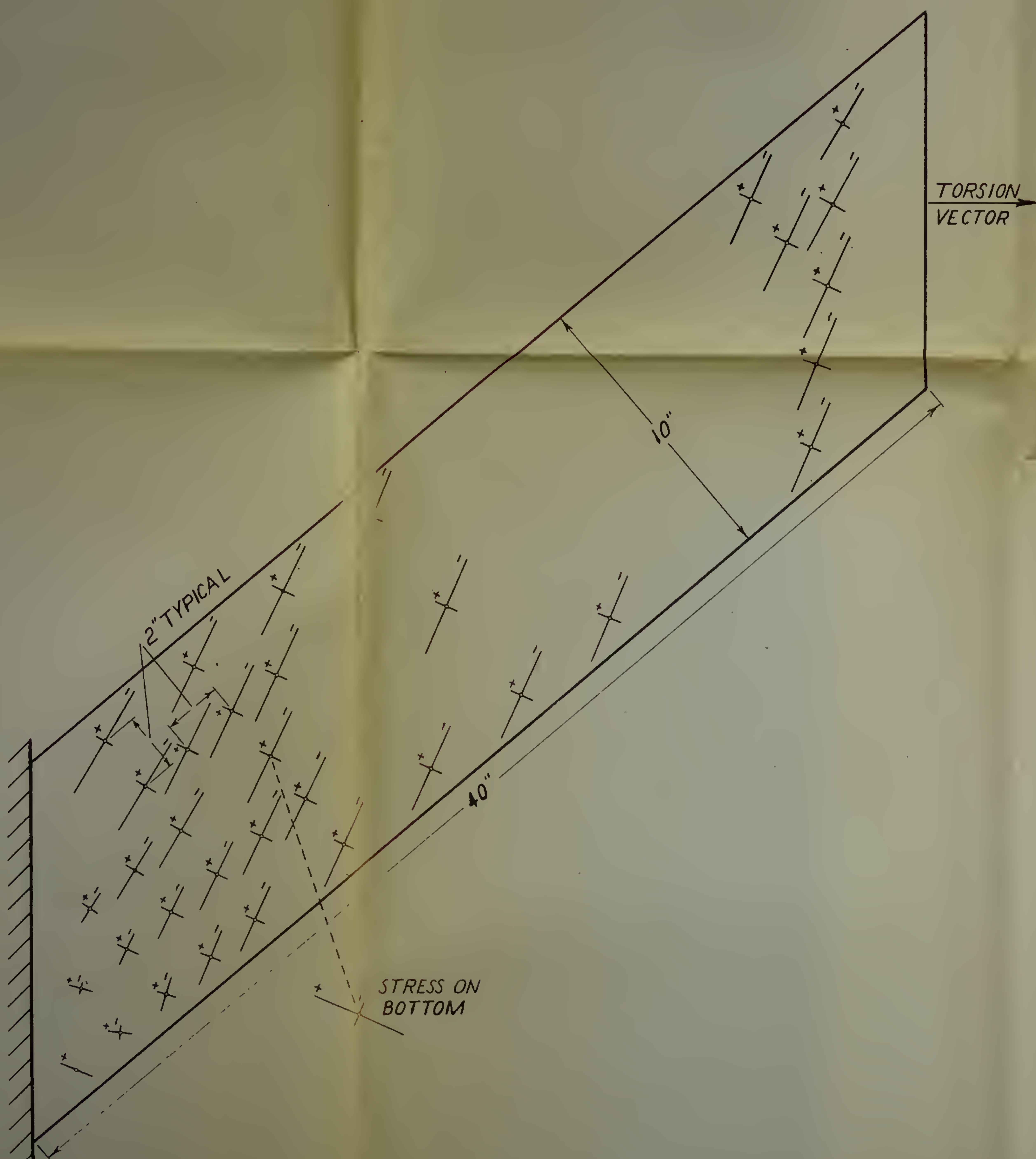


Figure 11

STRESSES PRODUCED BY
45000 IN-LBS TORSION VECTOR
PERPENDICULAR TO TIP
$\beta = 40°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION.
SCALE: 1" = 20 X 10³ PSI

MODEL #3 TEST #2

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

Figure 12

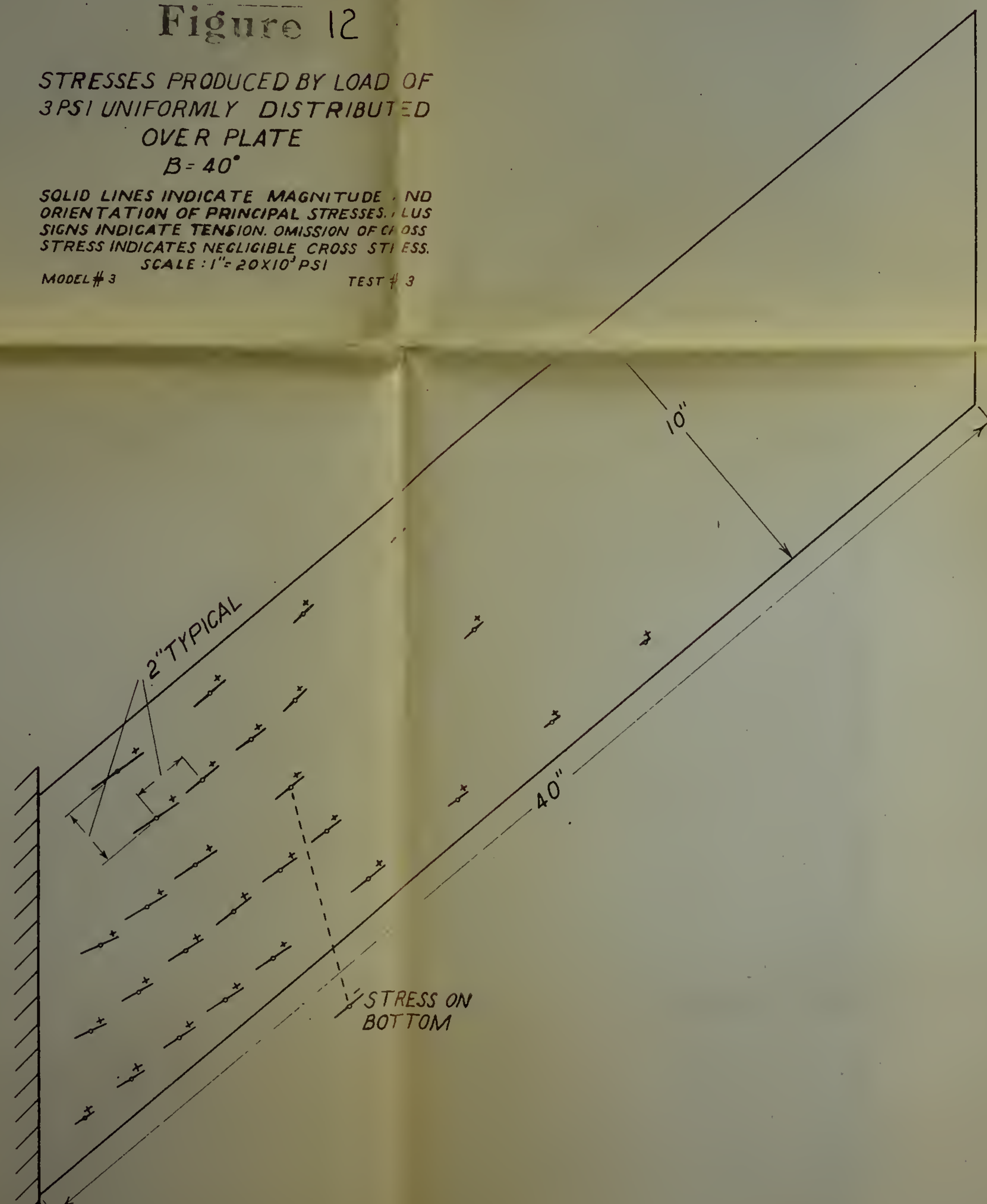
STRESSES PRODUCED BY LOAD OF 3 PSI UNIFORMLY DISTRIBUTED OVER PLATE

$B = 40^\circ$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS.

SCALE : 1" = 20×10^3 PSI

MODEL # 3 TEST # 3



PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

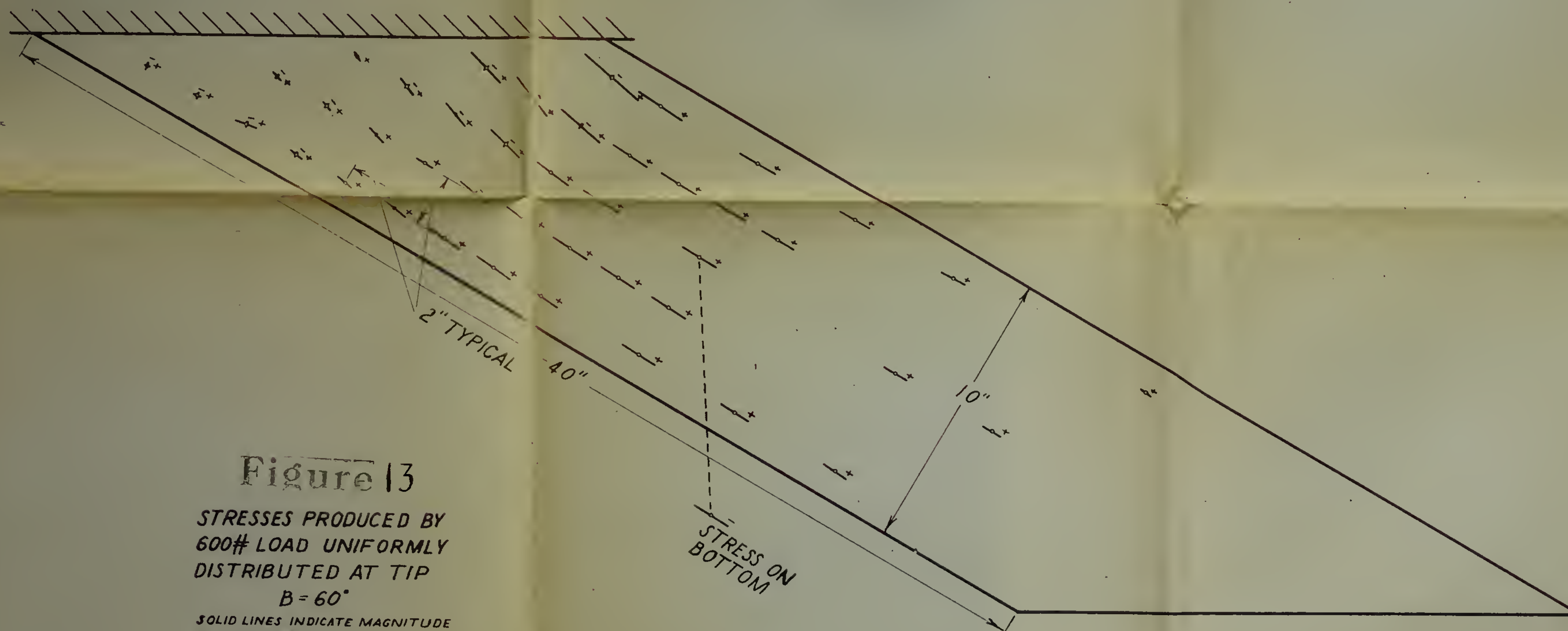


Figure 13

STRESSES PRODUCED BY 600# LOAD UNIFORMLY DISTRIBUTED AT TIP

B = 60°

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPLE STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20x10³ PSI

MODEL # 4 TEST # 1

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

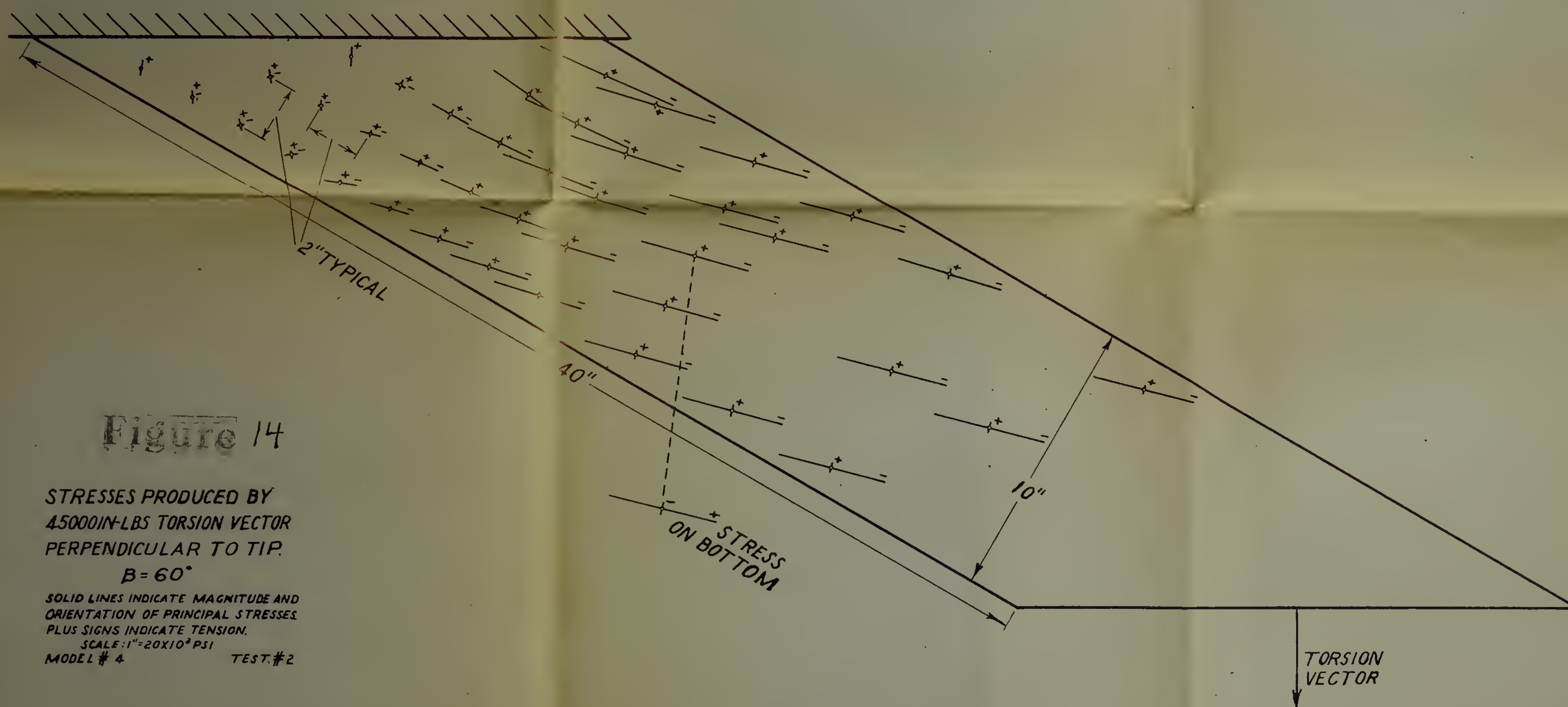


Figure 14

STRESSES PRODUCED BY
45000 IN-LBS TORSION VECTOR
PERPENDICULAR TO TIP.
$\beta = 60^\circ$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES.
PLUS SIGNS INDICATE TENSION.
SCALE: 1" = 20×10^3 PSI
MODEL # 4 TEST # 2

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

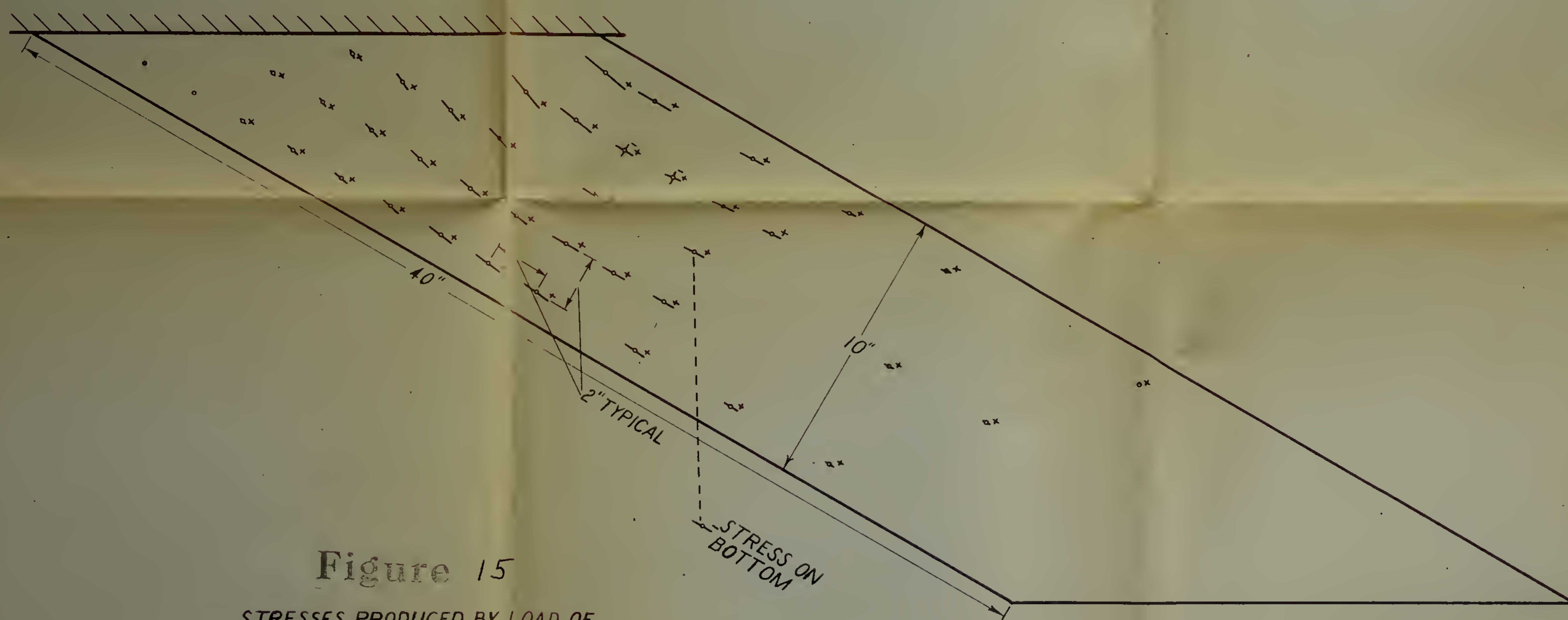


Figure 15

STRESSES PRODUCED BY LOAD OF 3 PSI UNIFORMLY DISTRIBUTED OVER PLATE

$B = 60°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS.

SCALE: 1" = 20X10³ PSI

MODEL # 4 TEST # 3

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

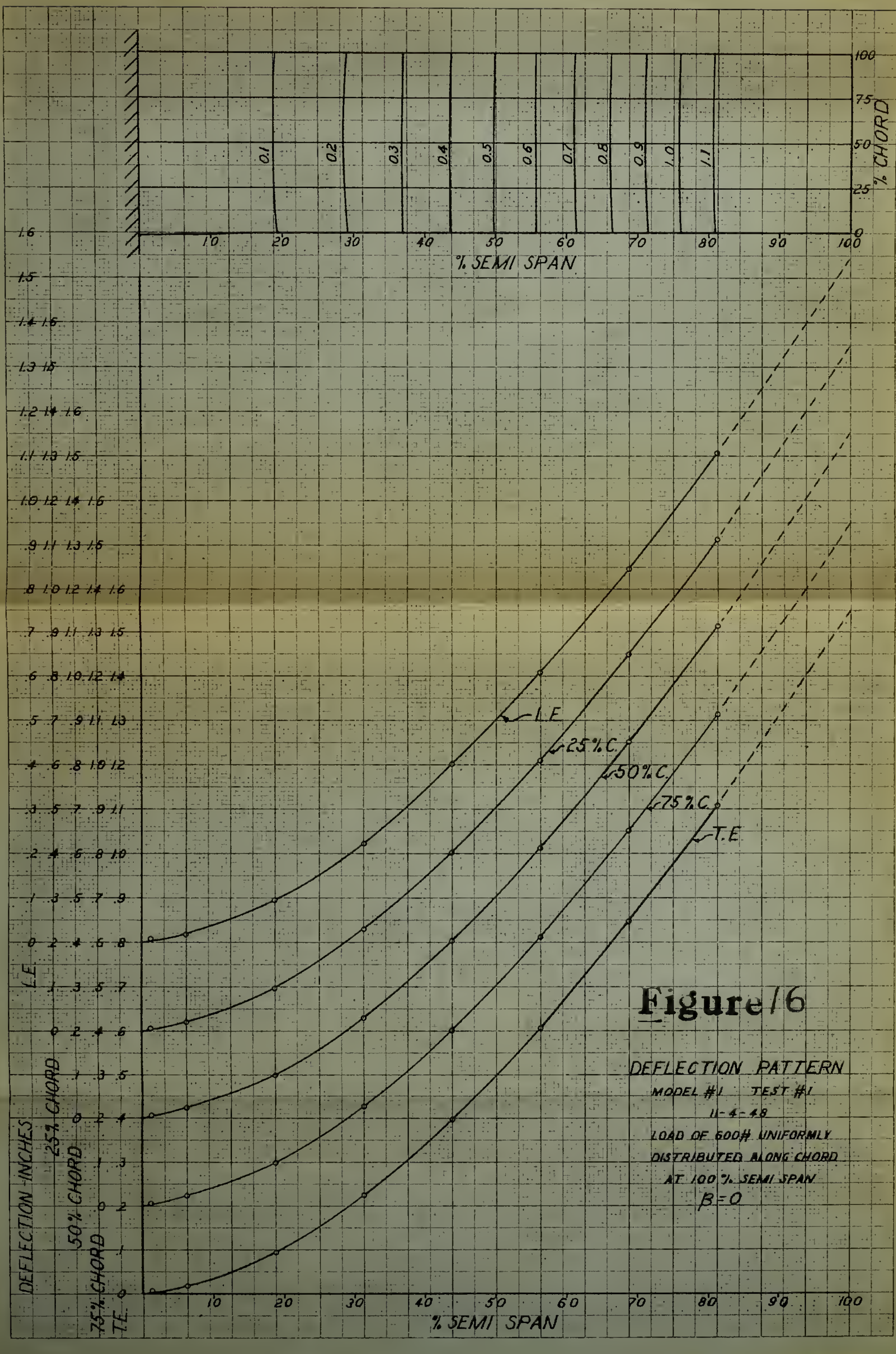


Figure 16

DEFLECTION PATTERN

MODEL #1 TEST #1

11-4-48

LOAD OF 600# UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

$\beta = 0$

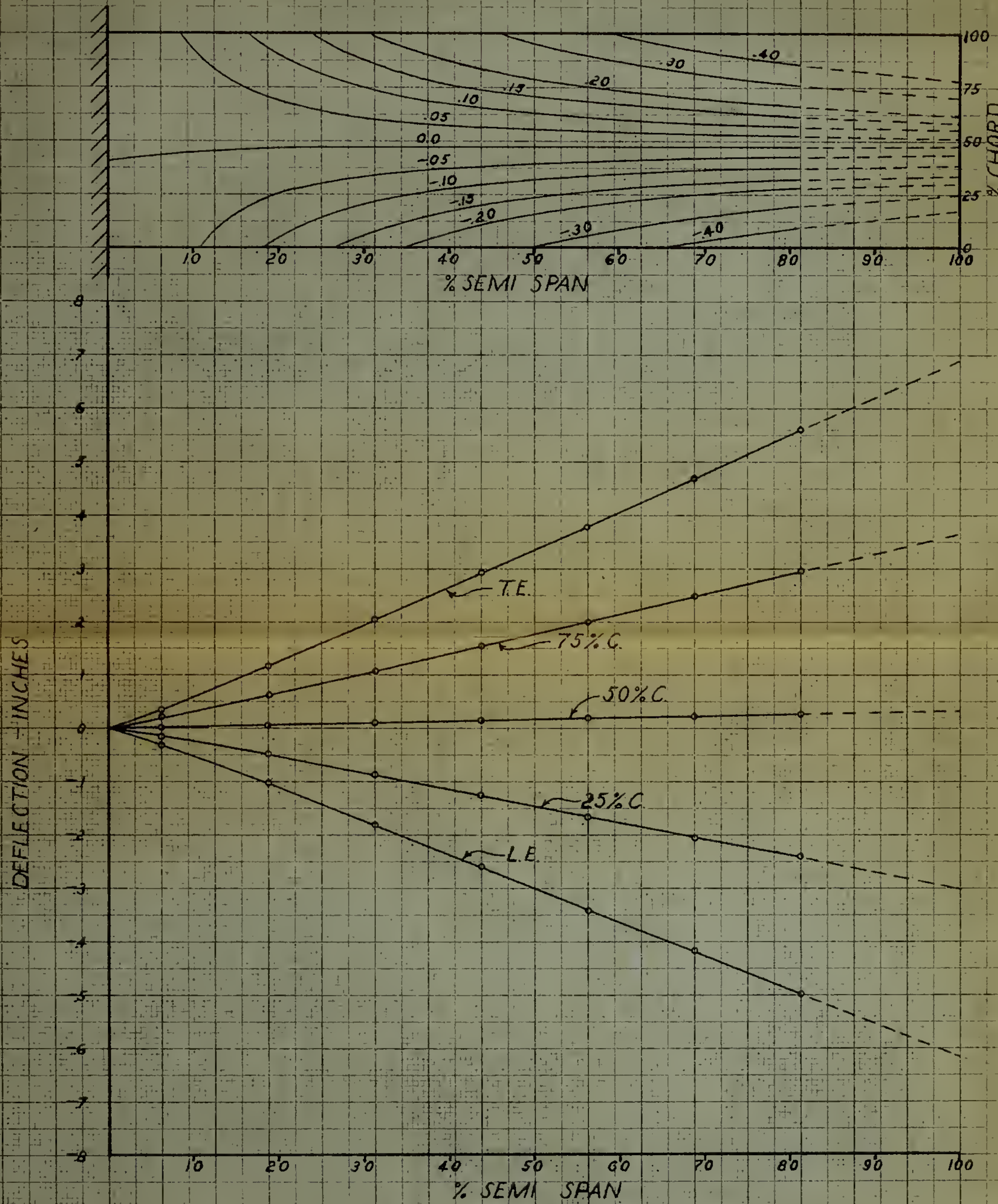


Figure 17

DEFLECTION PATTERN

MODEL #1 TEST #2

12-5-48

LOADED AT TIP WITH TORSIONAL MOMENT

VECTOR OF 45000 IN-# PERPENDICULAR TO ROOT

B=0°

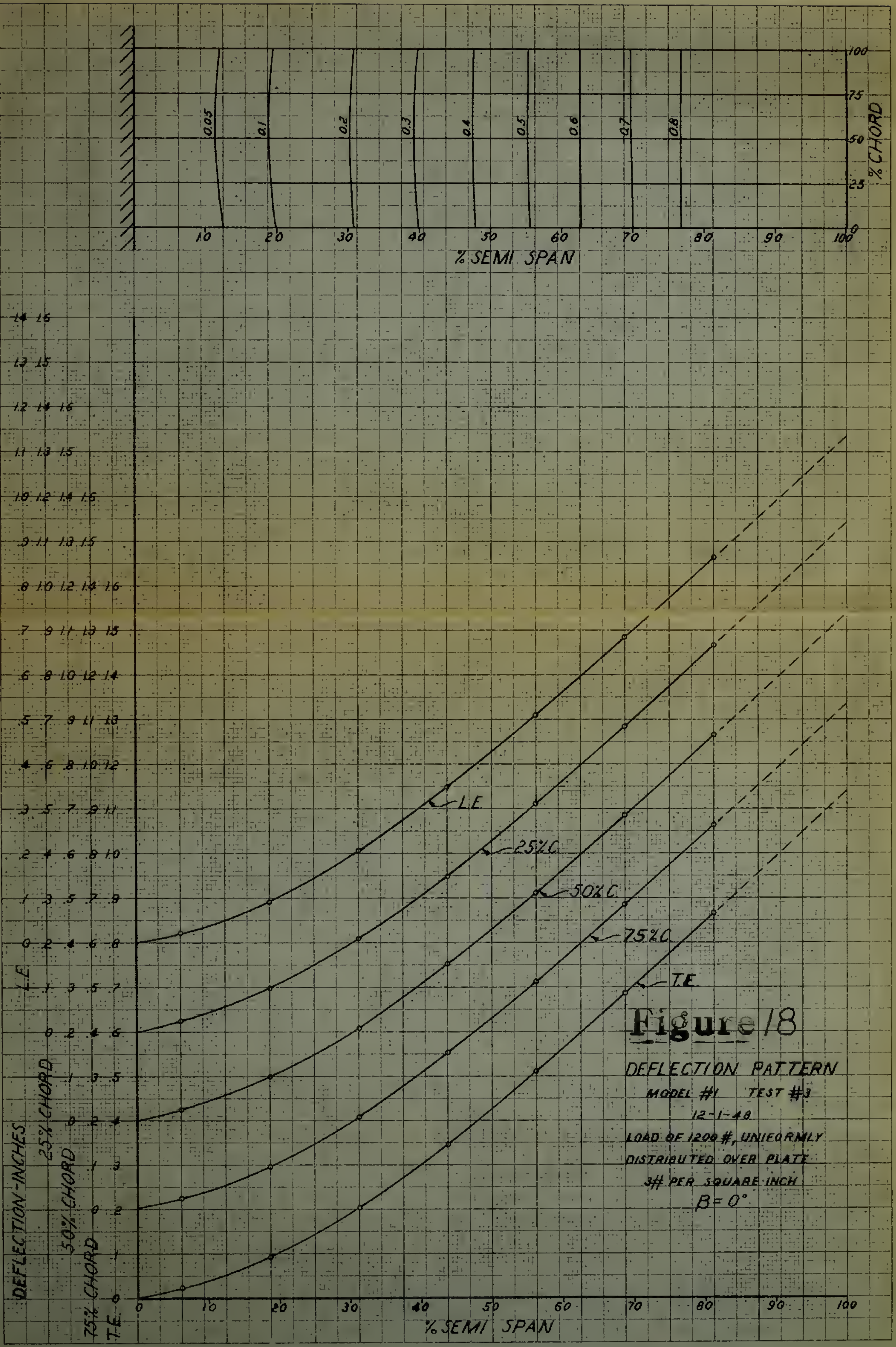


Figure 18

DEFLECTION PATTERN

MODEL #1 TEST #3

12-1-48

LOAD OF 1200 #, UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE INCH

$\beta = 0°$

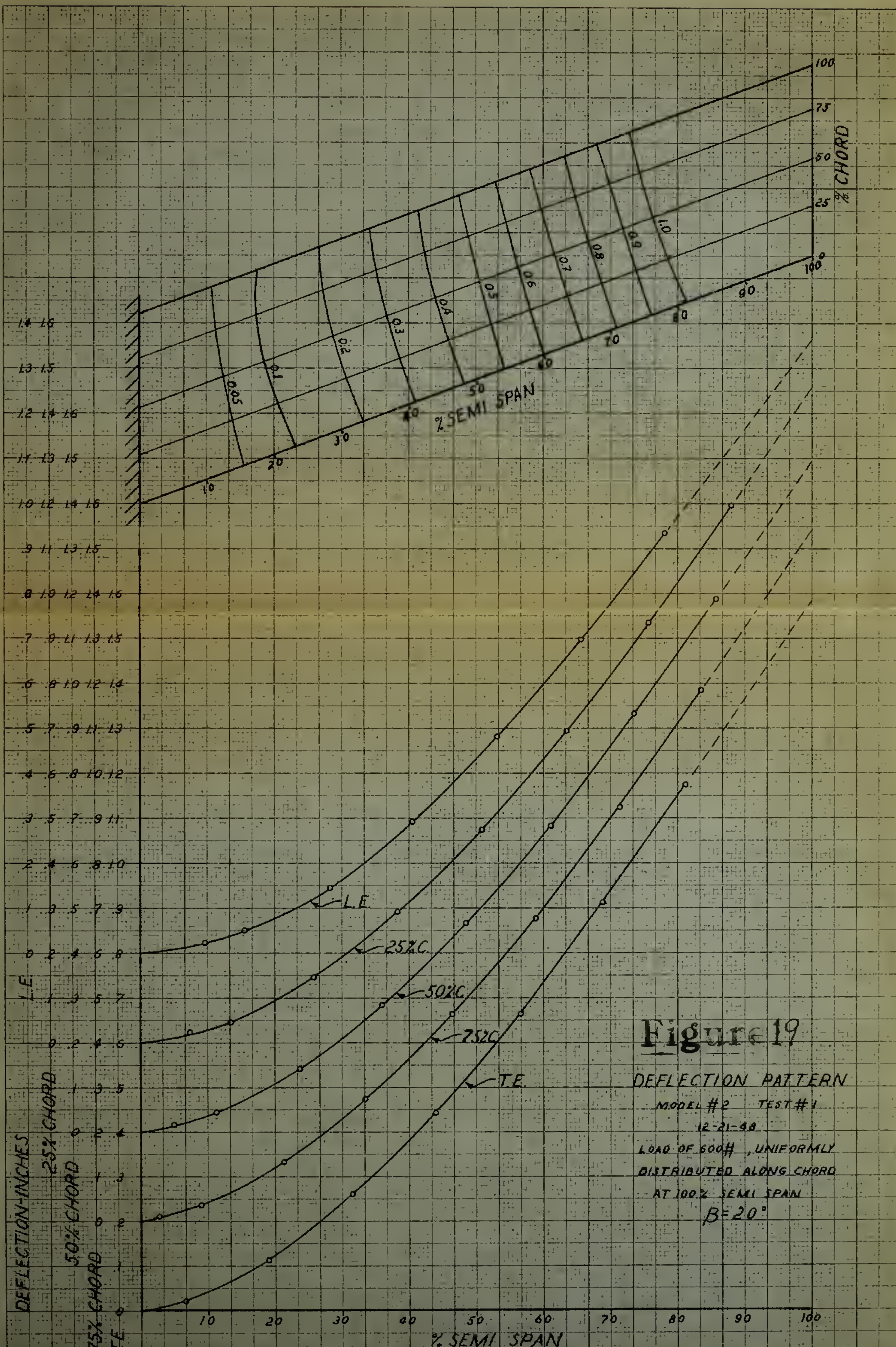


Figure 19

DEFLECTION PATTERN

MODEL #2 TEST #1

12-21-48

LOAD OF 600# , UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

$\beta = 20°$

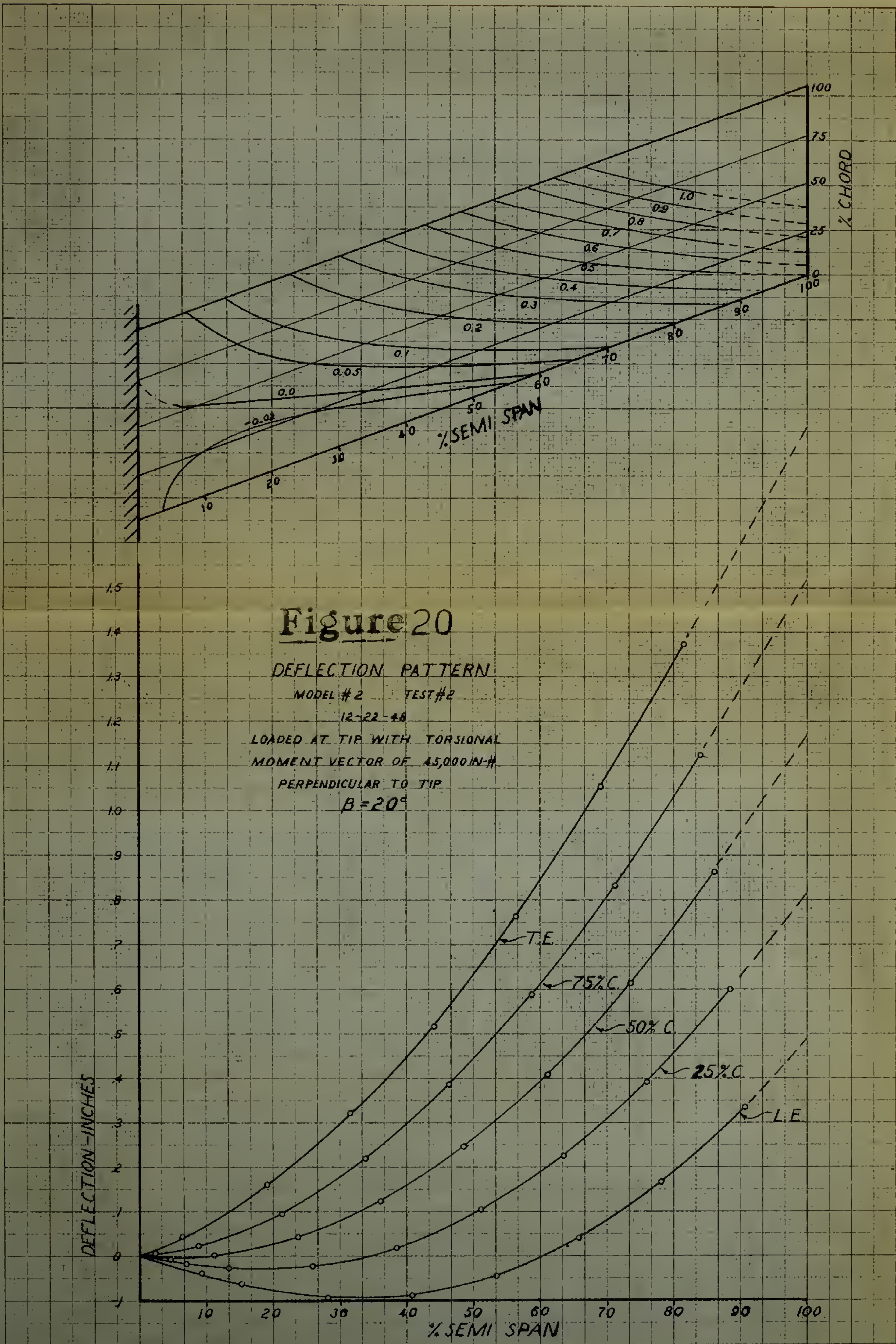


Figure 20
DEFLECTION PATTERN
MODEL #2 TEST #2
12-22-48
LOADED AT TIP WITH TORSIONAL
MOMENT VECTOR OF 45,000 IN-#
PERPENDICULAR TO TIP
β = 20°
% CHORD
% SEMI SPAN
DEFLECTION-INCHES
T.E.
75% C.
50% C.
25% C.
L.E.

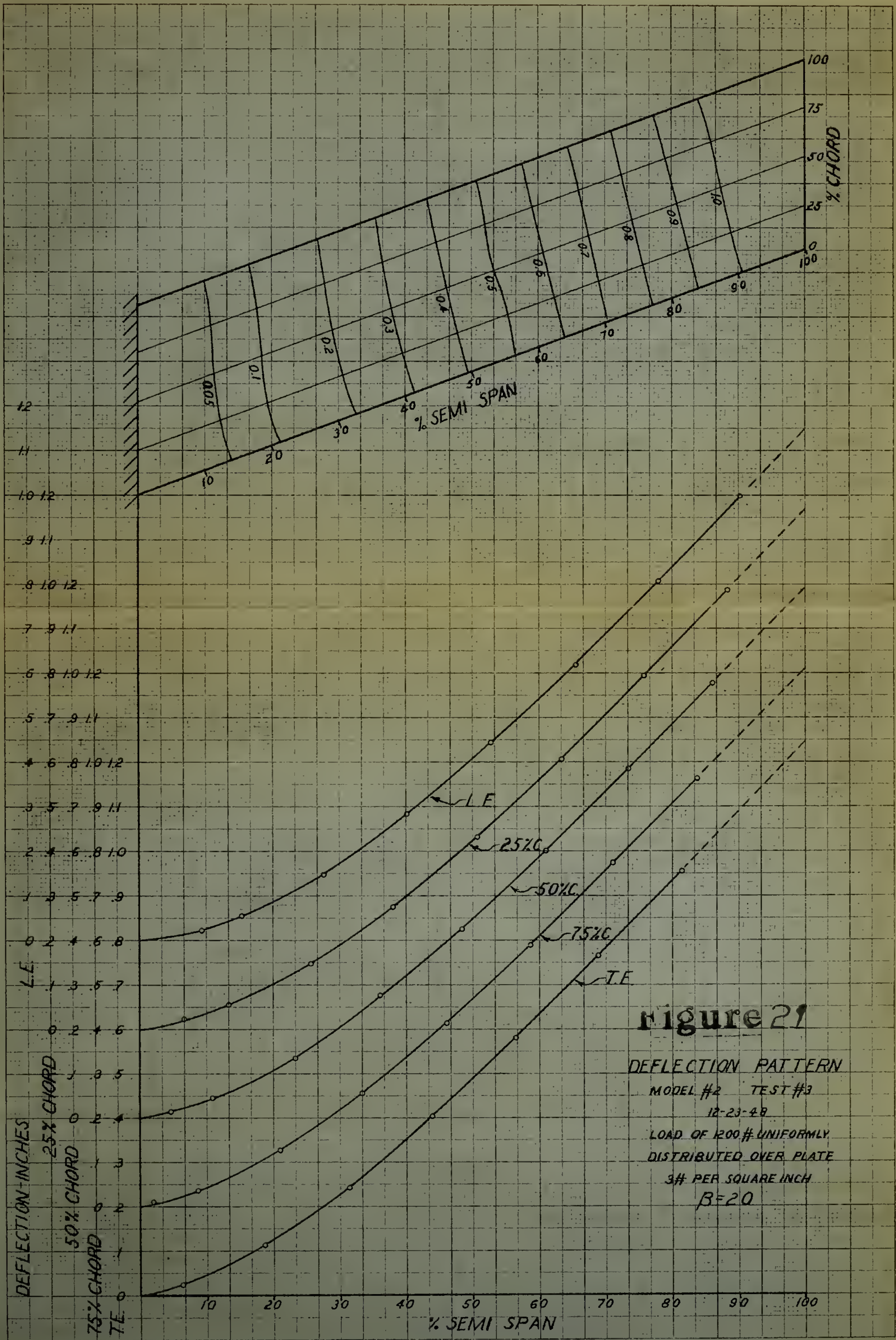


Figure 21

DEFLECTION PATTERN

MODEL #2 TEST #3

12-23-48

LOAD OF 1200 # UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE INCH

$\beta = 20$

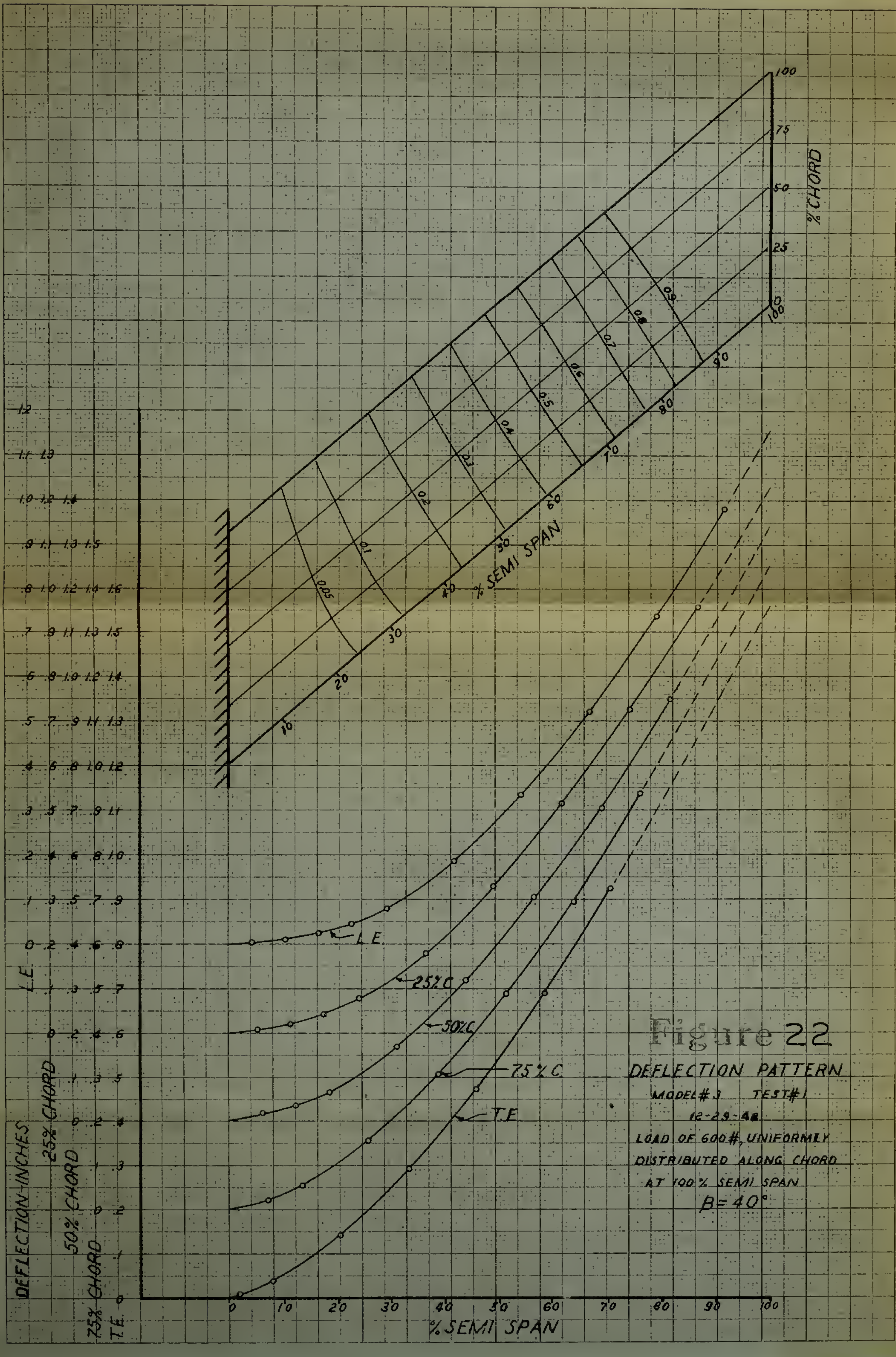


Figure 22

DEFLECTION PATTERN
MODEL #3 TEST #1
12-29-48
LOAD OF 600#, UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN
B = 40°

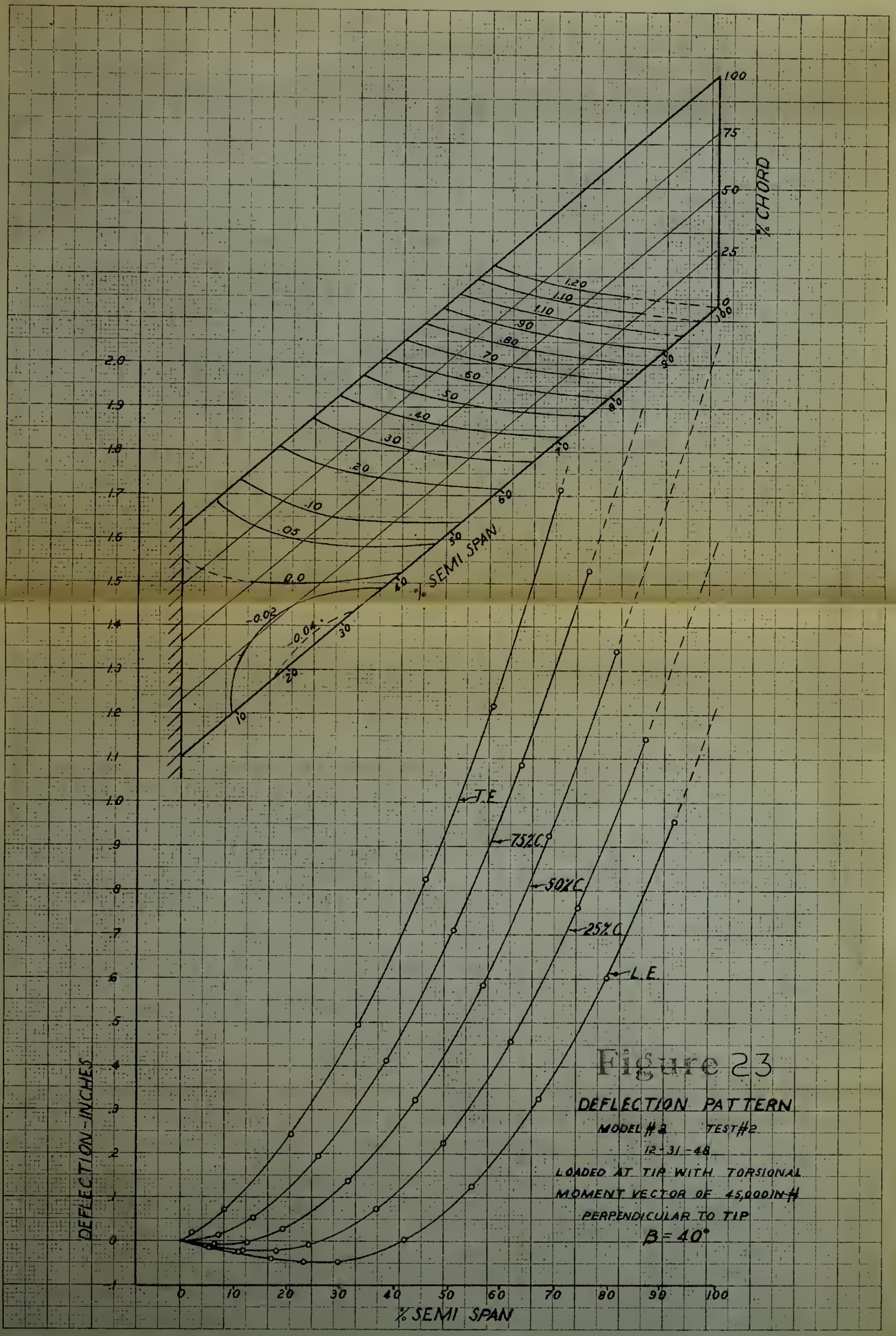


Figure 23

DEFLECTION PATTERN

MODEL #2 TEST #2

12-31-48

LOADED AT TIP WITH TORSIONAL MOMENT VECTOR OF 45,000 IN # PERPENDICULAR TO TIP

$\beta = 40°$

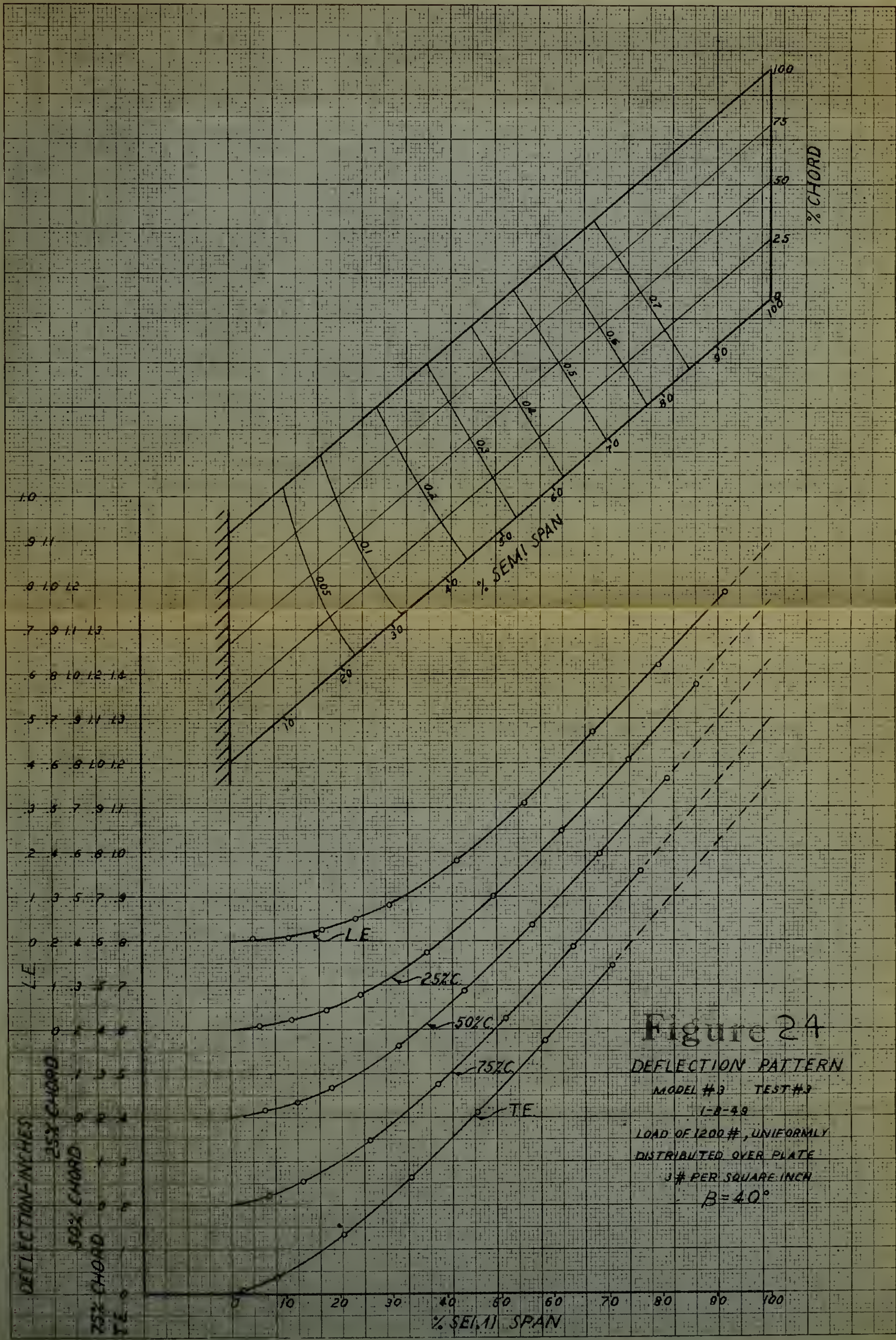


Figure 24

DEFLECTION PATTERN

MODEL #3 TEST #3

1-8-49

LOAD OF 1200 #, UNIFORMLY DISTRIBUTED OVER PLATE

3 # PER SQUARE INCH

$\beta = 40°$

Figure 25

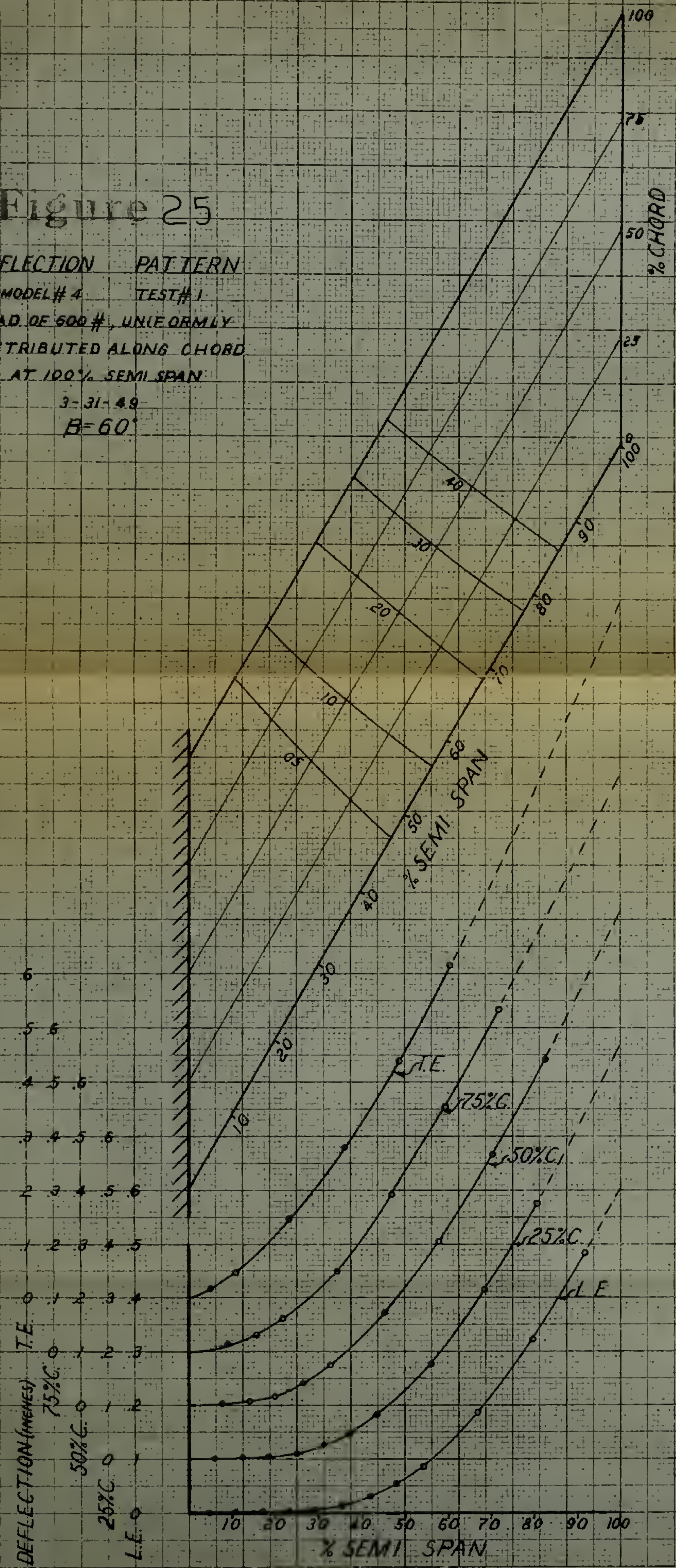
DEFLECTION PATTERN

MODEL # 4 TEST # 1

LOAD OF 600 #, UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

3-31-49

B=60°



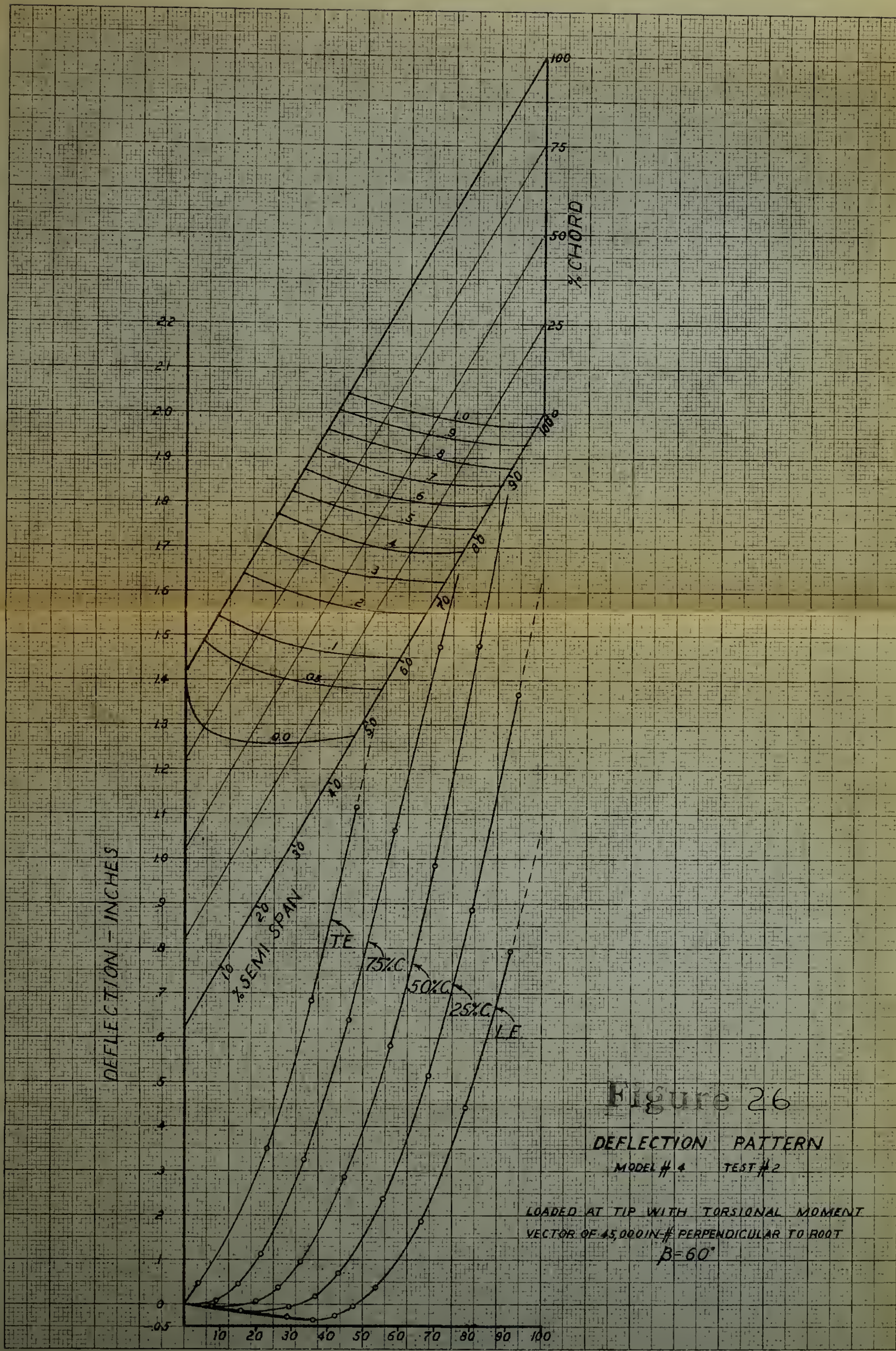


Figure 26

DEFLECTION PATTERN

MODEL # 4 TEST # 2

LOADED AT TIP WITH TORSIONAL MOMENT VECTOR OF 45,000 IN-# PERPENDICULAR TO ROOT

$\beta = 60°$

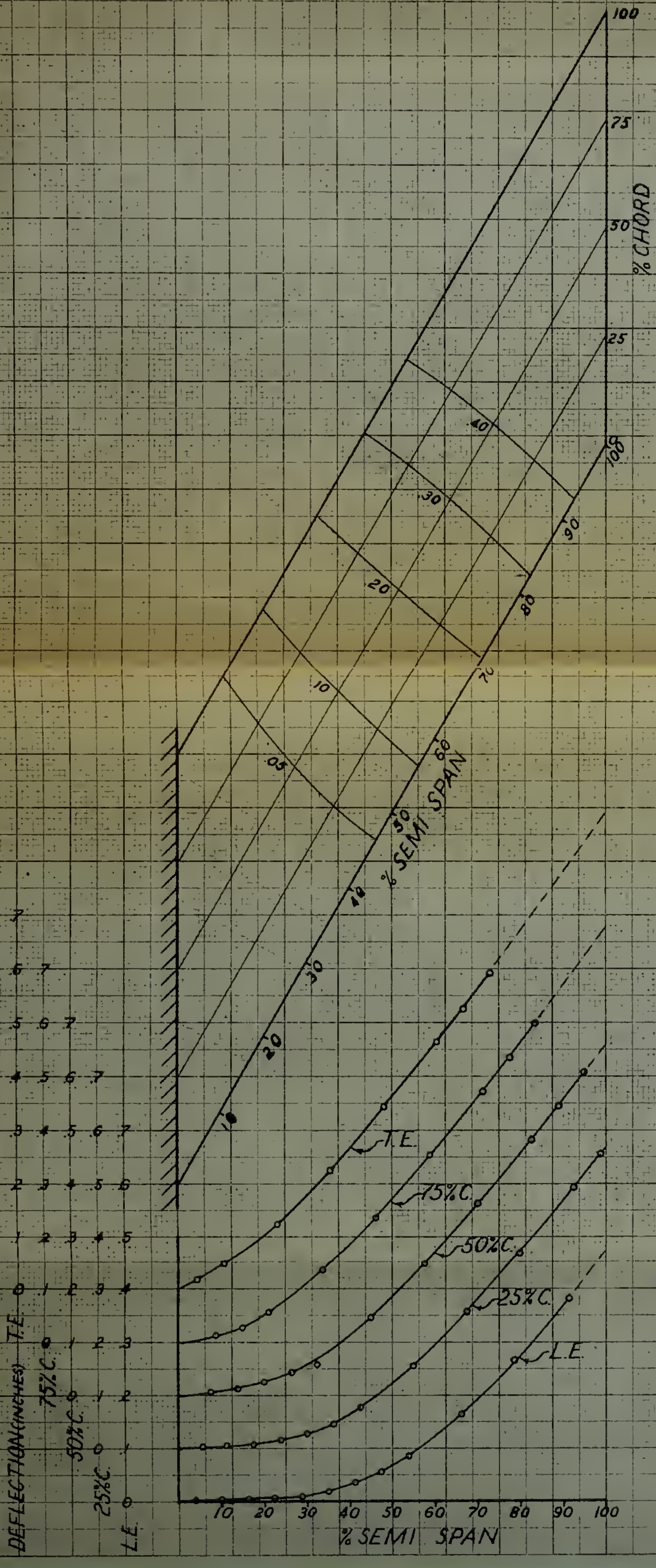


Figure 27

DEFLECTION PATTERN

MODEL # 4 TEST # 3

3-30-49

LOAD OF 1200#, UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE IN.

$\beta = 60°$

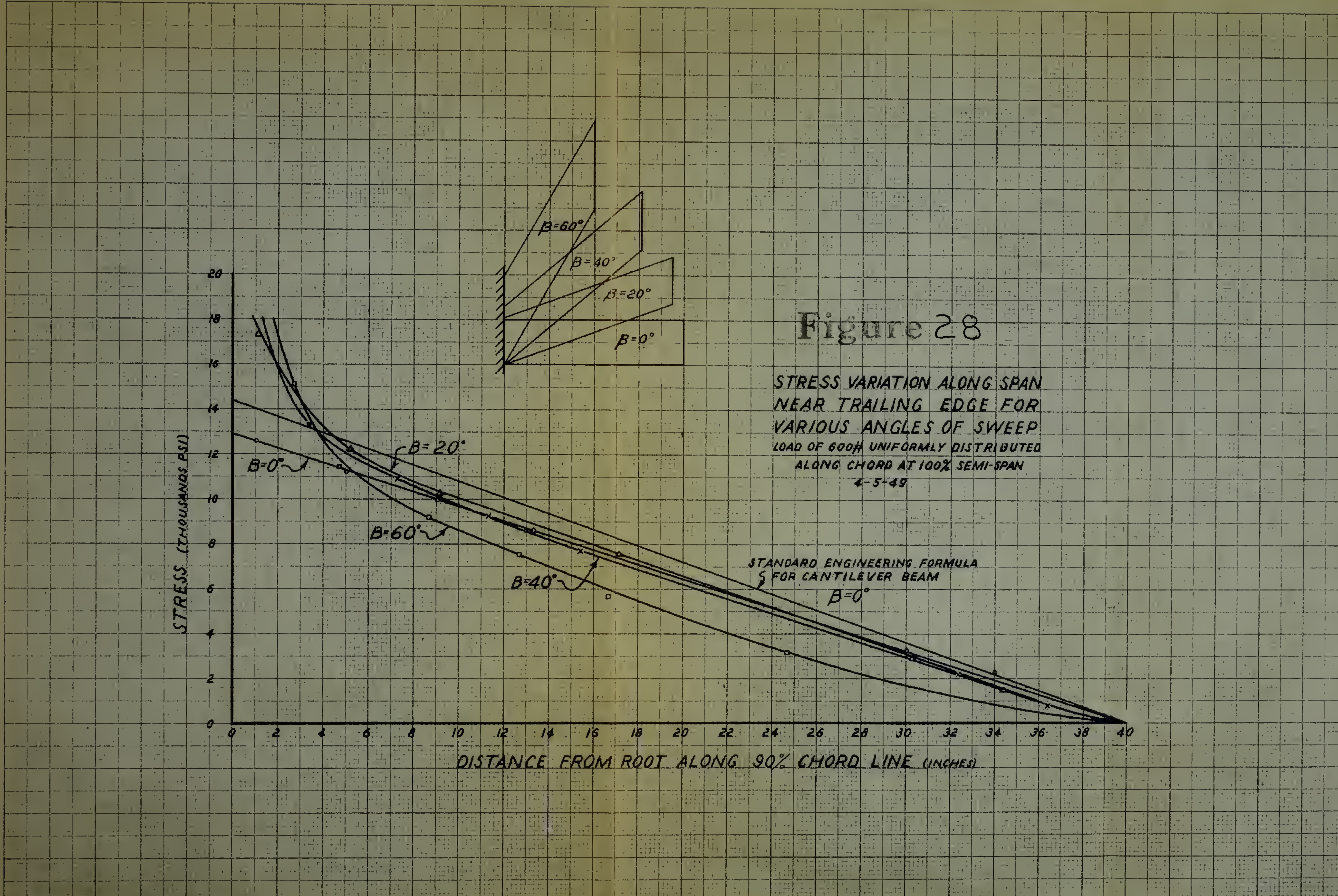

β=60°
β=40°
β=20°
β=0°
STRESS VARIATION ALONG SPAN
NEAR TRAILING EDGE FOR
VARIOUS ANGLES OF SWEEP
LOAD OF 600# UNIFORMLY DISTRIBUTED
ALONG CHORD AT 100% SEMI-SPAN
4-5-49
B=20°
B=0°
B=60°
B=40°
STANDARD ENGINEERING FORMULA
FOR CANTILEVER BEAM
β=0°
STRESS (THOUSANDS PSI)
0
2
4
6
8
10
12
14
16
18
20
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
DISTANCE FROM ROOT ALONG 90% CHORD LINE (INCHES)

Figure 28

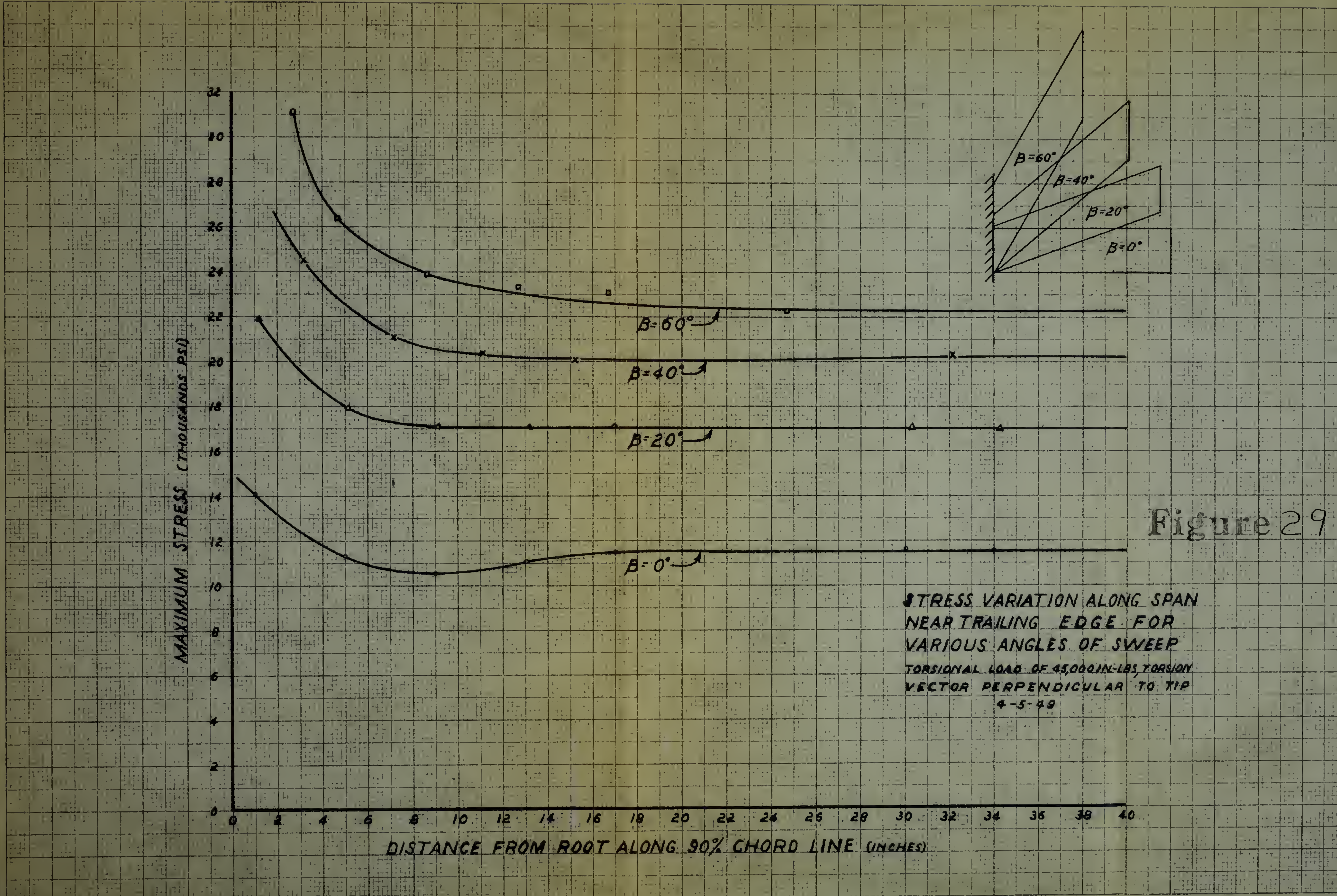


Figure 29

STRESS VARIATION ALONG SPAN NEAR TRAILING EDGE FOR VARIOUS ANGLES OF SWEEP

TORSIONAL LOAD OF 45,000 IN-LBS, TORSION VECTOR PERPENDICULAR TO TIP

4-5-49

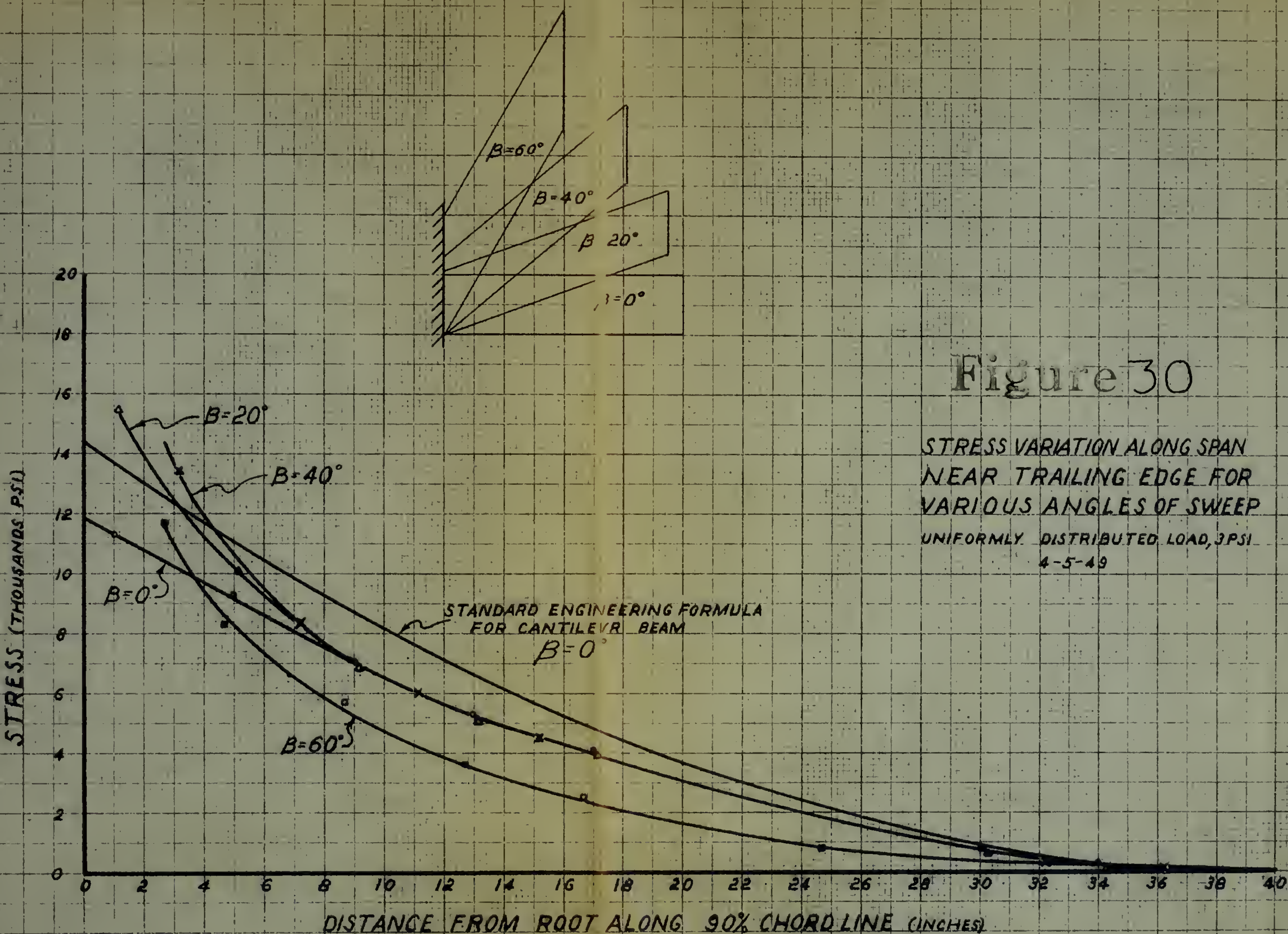


Figure 30

STRESS VARIATION ALONG SPAN
NEAR TRAILING EDGE FOR
VARIOUS ANGLES OF SWEEP

UNIFORMLY DISTRIBUTED LOAD, 3 PSI
4-5-49

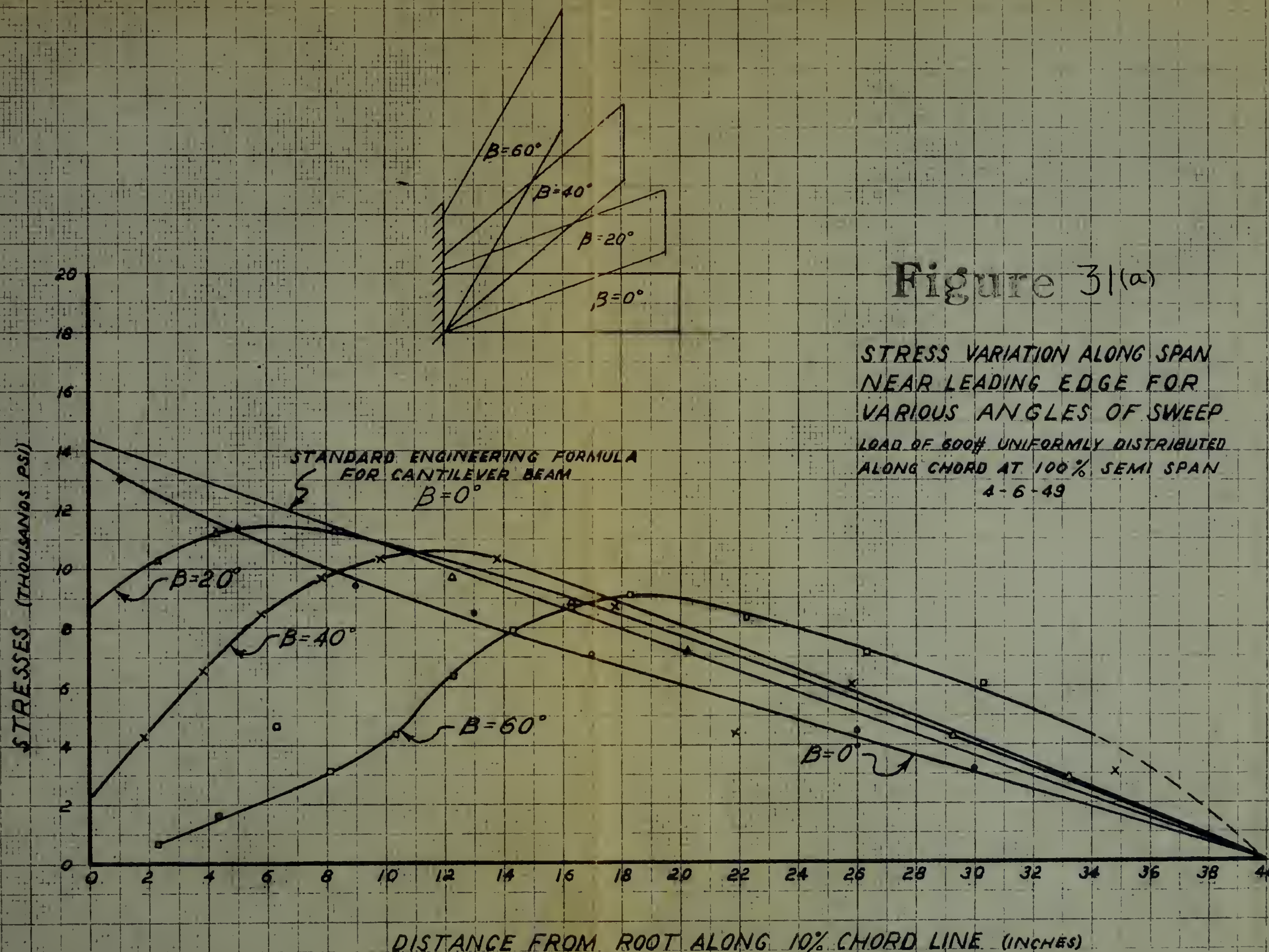


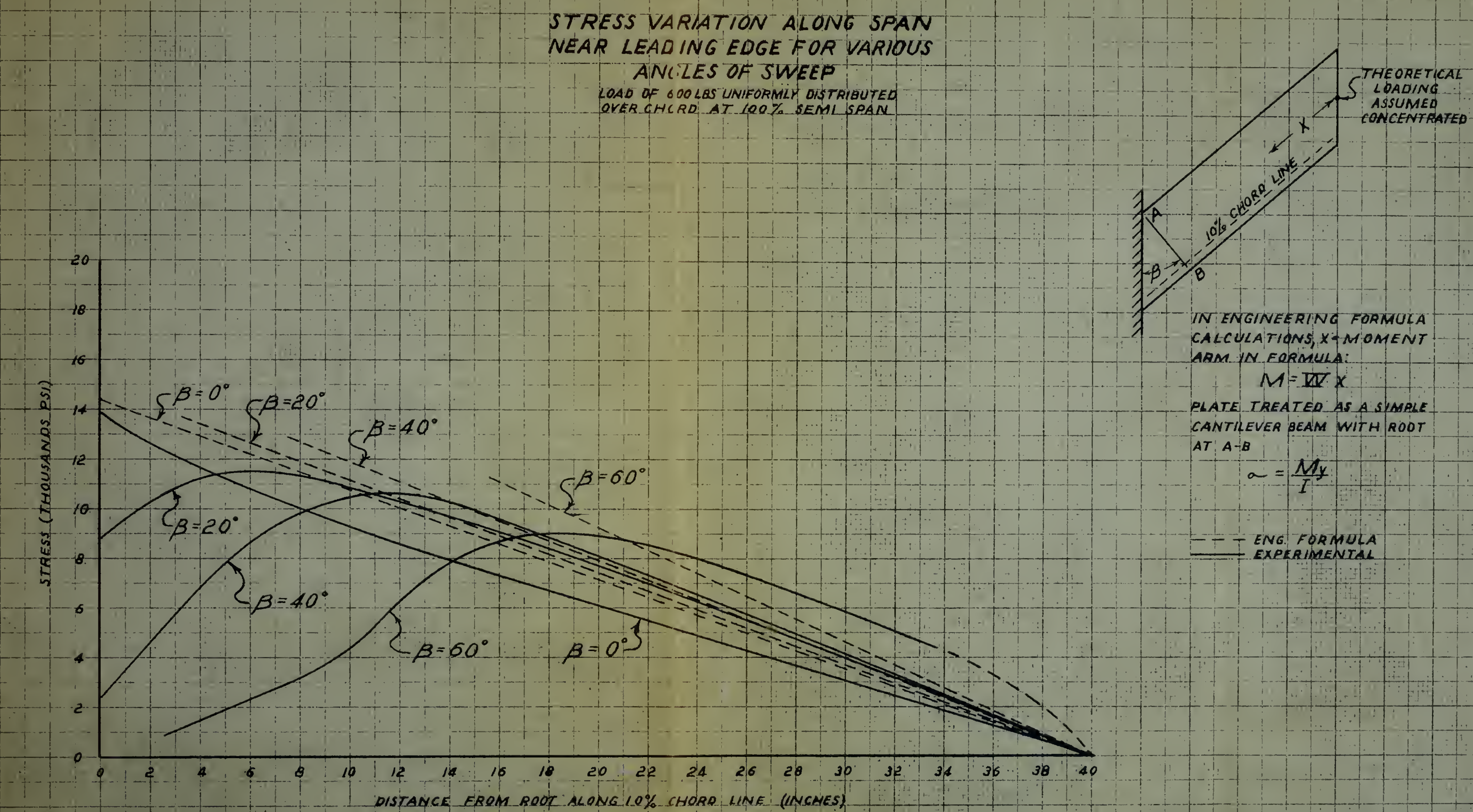
Figure 31(a)

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

LOAD OF 600# UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

4-6-49

Figure 31(b)



STRESS VARIATION ALONG SPAN
NEAR LEADING EDGE FOR VARIOUS
ANGLES OF SWEEP
LOAD OF 600 LBS UNIFORMLY DISTRIBUTED
OVER CHORD AT 100% SEMI SPAN
THEORETICAL LOADING ASSUMED CONCENTRATED
X
10% CHORD LINE
A
B
β
IN ENGINEERING FORMULA CALCULATIONS, X = MOMENT ARM IN FORMULA:
M = W X
PLATE TREATED AS A SIMPLE CANTILEVER BEAM WITH ROOT AT A-B
σ = My/I
ENG. FORMULA
EXPERIMENTAL
STRESS (THOUSANDS PSI)
0
2
4
6
8
10
12
14
16
18
20
β = 0°
β = 20°
β = 40°
β = 60°
β = 20°
β = 40°
β = 60°
β = 0°
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
DISTANCE FROM ROOT ALONG 10% CHORD LINE (INCHES)

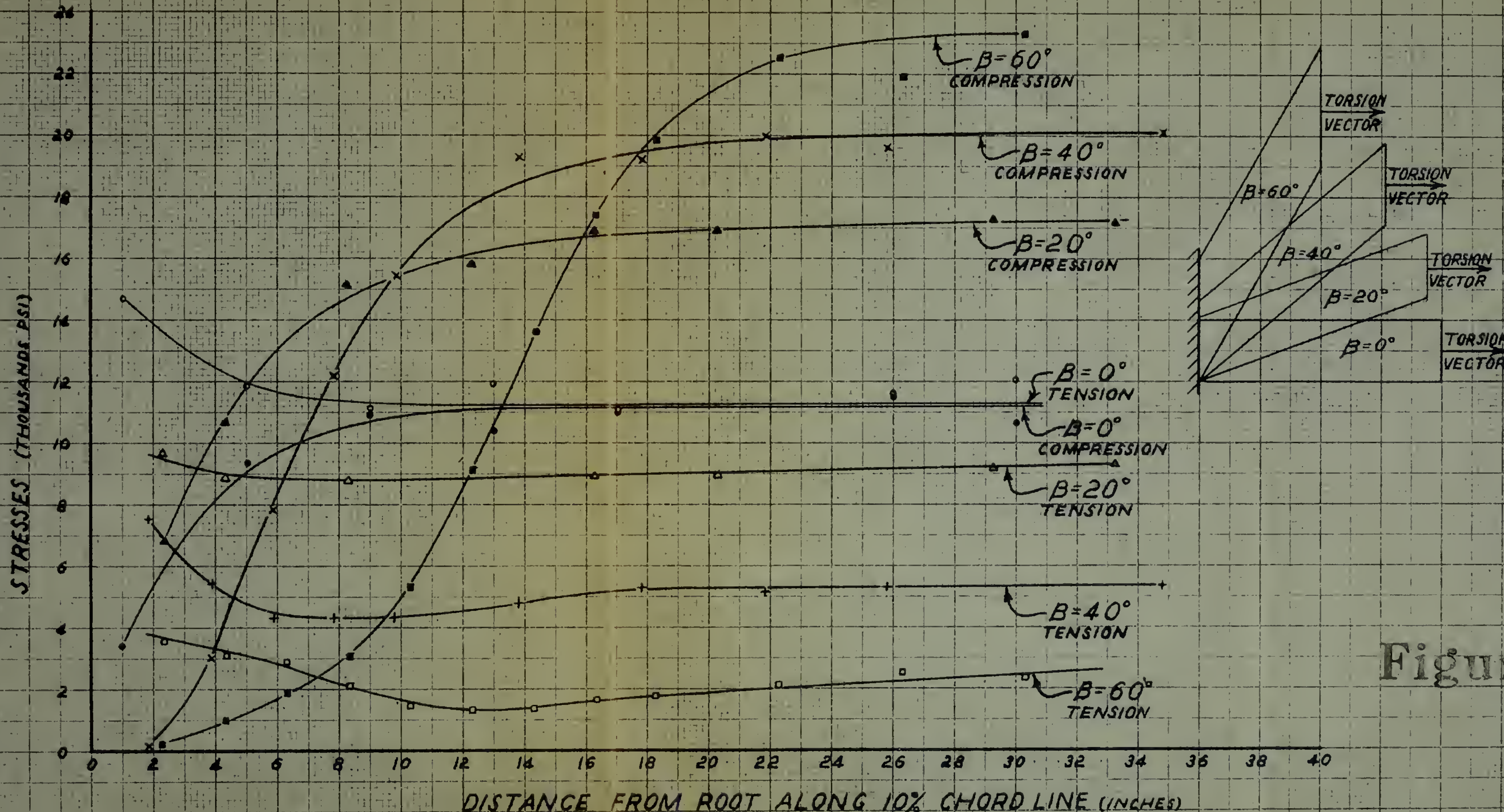


Figure 32

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

TORSIONAL LOAD OF 45,000 IN-LBS, TORSION VECTOR PERPENDICULAR TO TIP

4-5-49

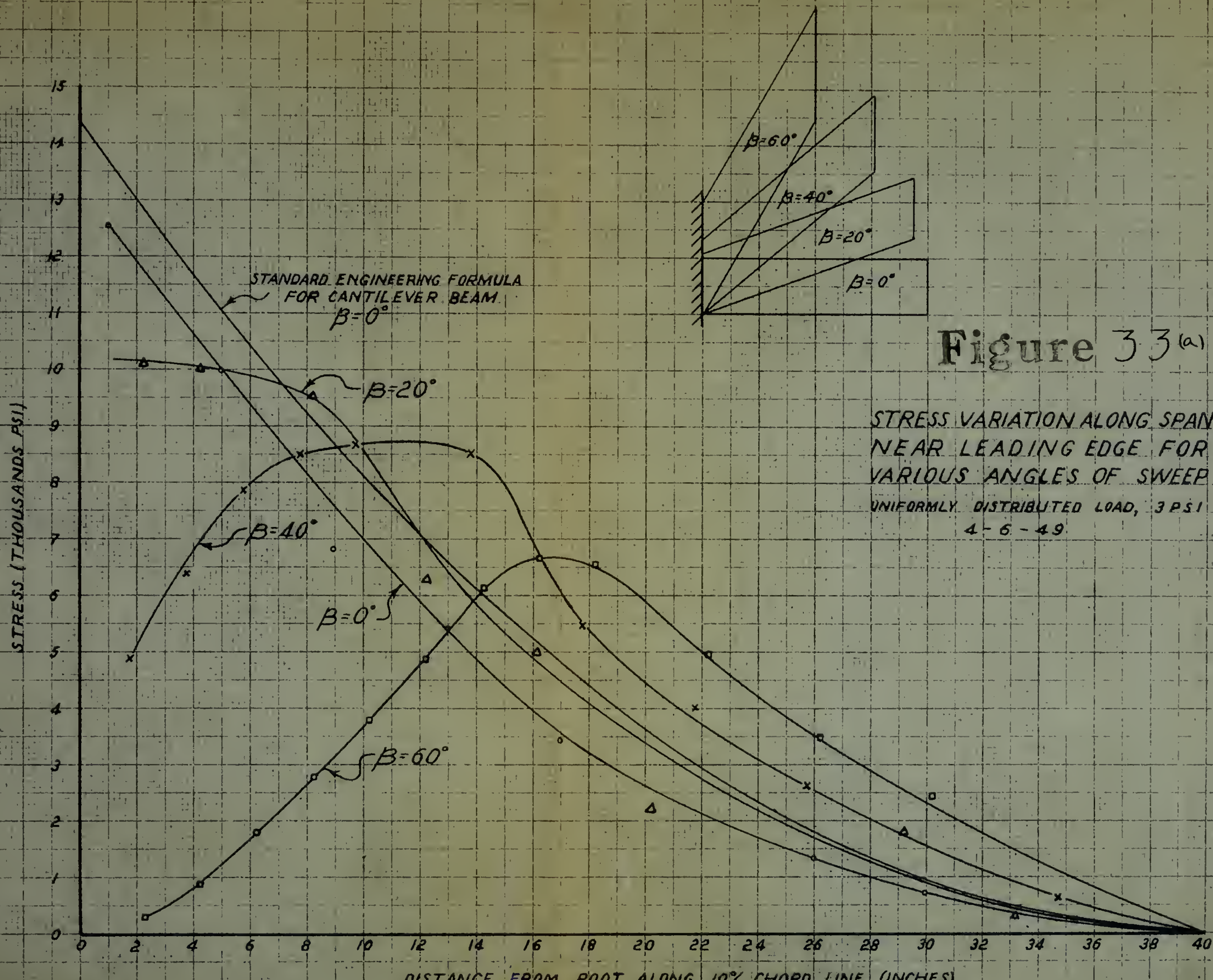


Figure 33(a)

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

UNIFORMLY DISTRIBUTED LOAD, 3 PSI

4 - 6 - 49

Figure 33(b)

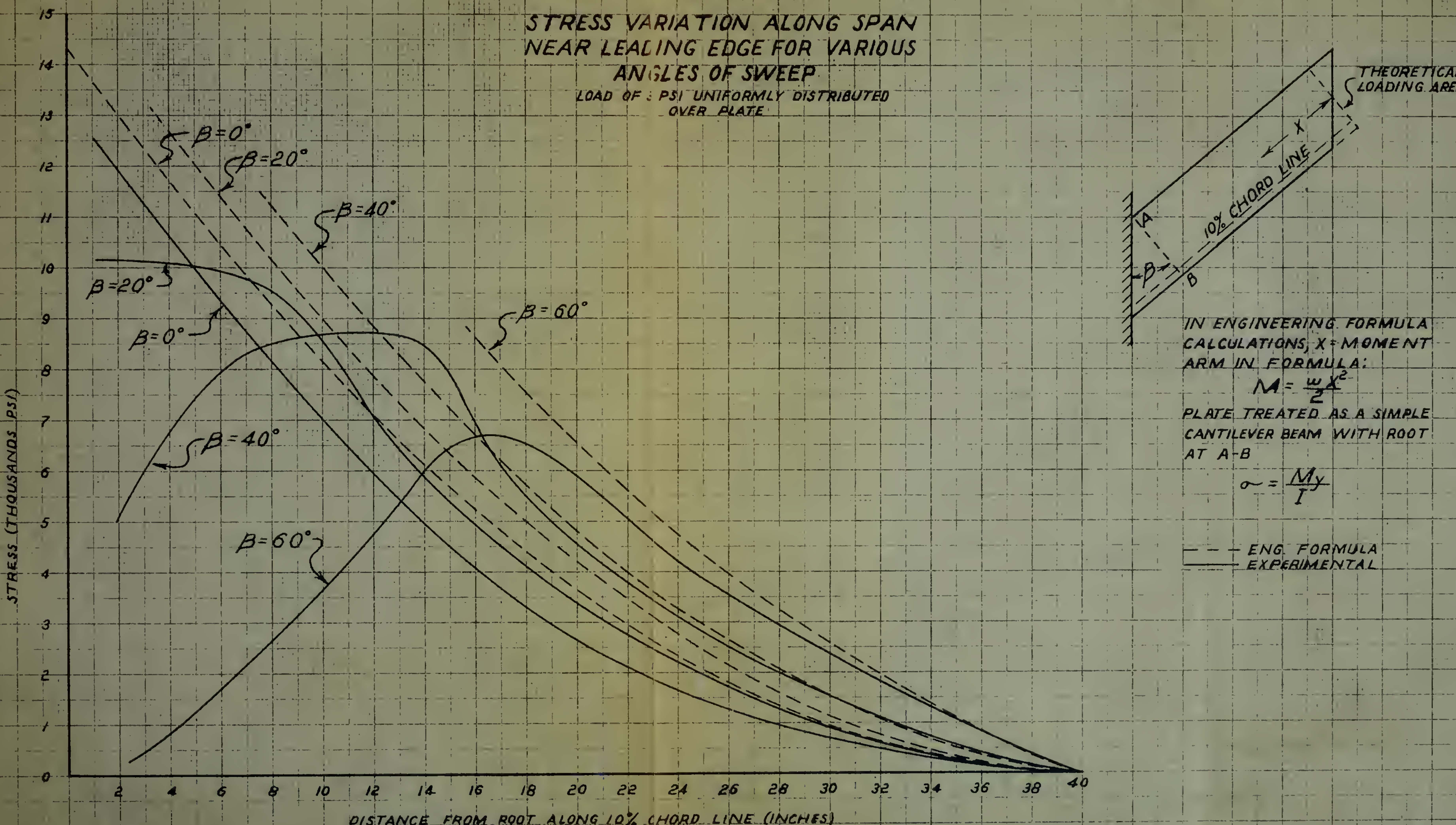
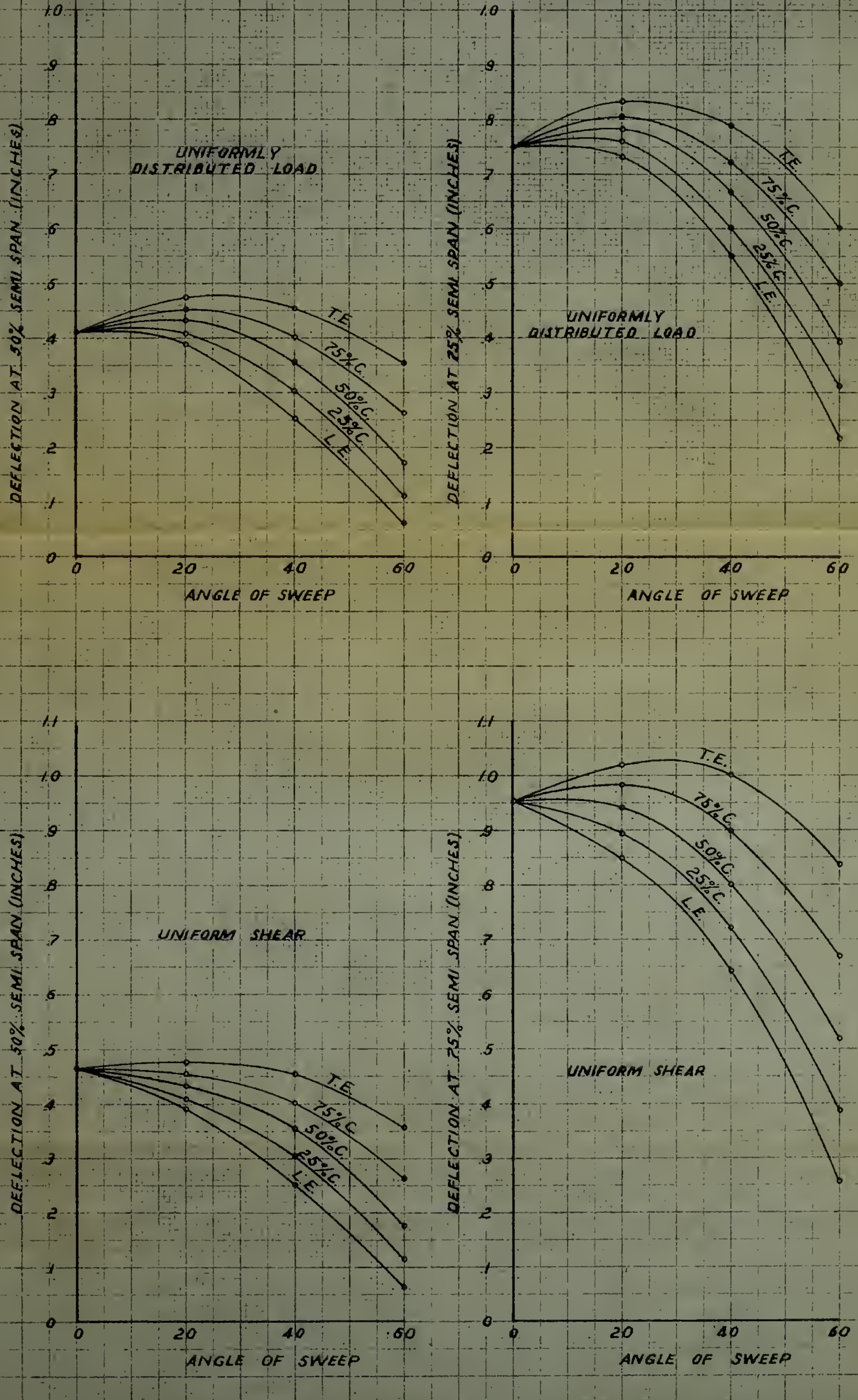

STRESS VARIATION ALONG SPAN
NEAR LEADING EDGE FOR VARIOUS
ANGLES OF SWEEP
LOAD OF 3 PSI UNIFORMLY DISTRIBUTED OVER PLATE
STRESS (THOUSANDS PSI)
DISTANCE FROM ROOT ALONG 10% CHORD LINE (INCHES)
β=0°
β=20°
β=40°
β=60°
THEORETICAL LOADING AREA
X
10% CHORD LINE
A
B
β
IN ENGINEERING FORMULA CALCULATIONS, X = MOMENT ARM IN FORMULA:
M = wX²/2
PLATE TREATED AS A SIMPLE CANTILEVER BEAM WITH ROOT AT A-B
σ = My/I
ENG. FORMULA
EXPERIMENTAL

Figure 34

DEFLECTION vs ANGLE OF SWEEP

UNIFORM SHEAR AND UNIFORMLY DISTRIBUTED LOADS

4-6-49



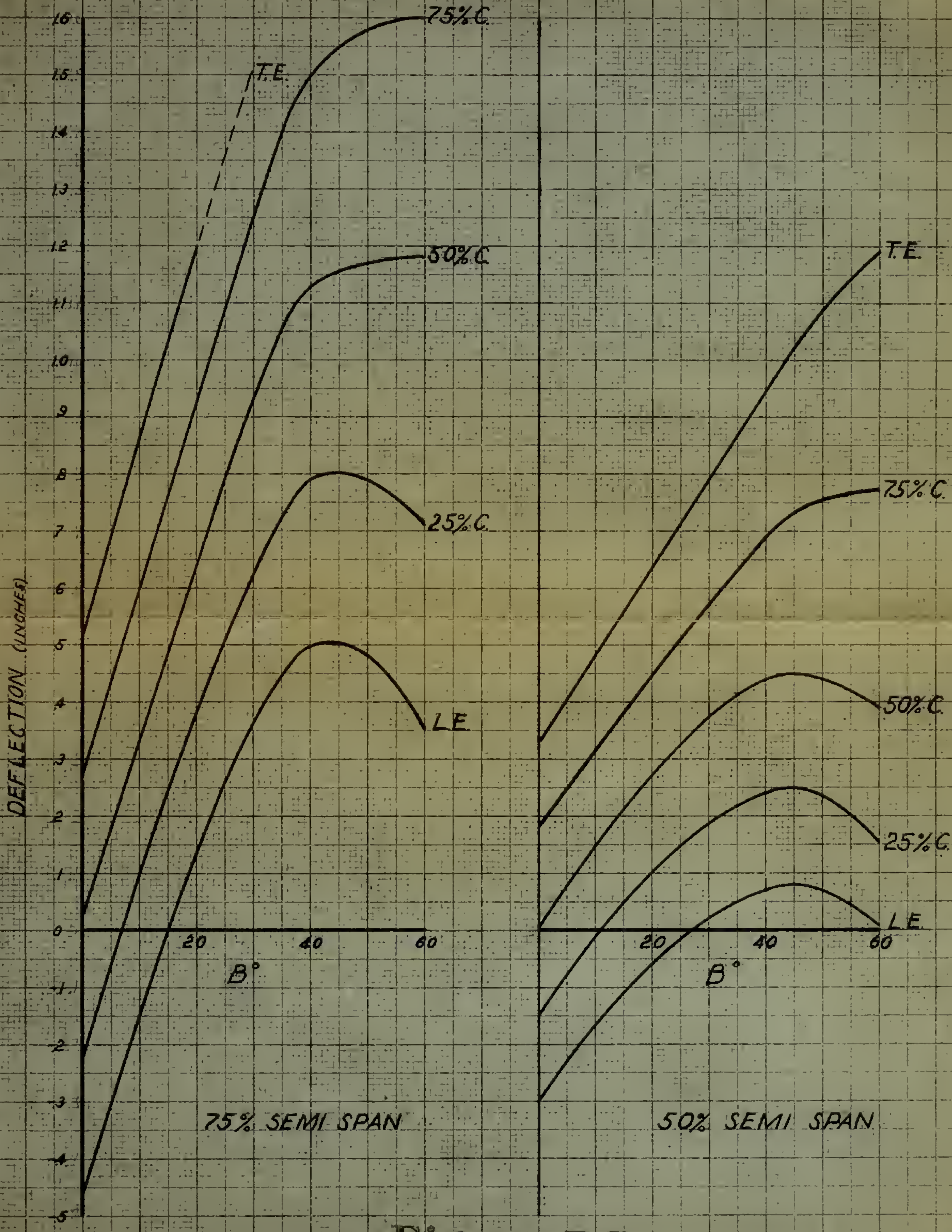


Figure 35

DEFLECTION vs ANGLE OF SWEEP
TORSION LOAD
4-6-49

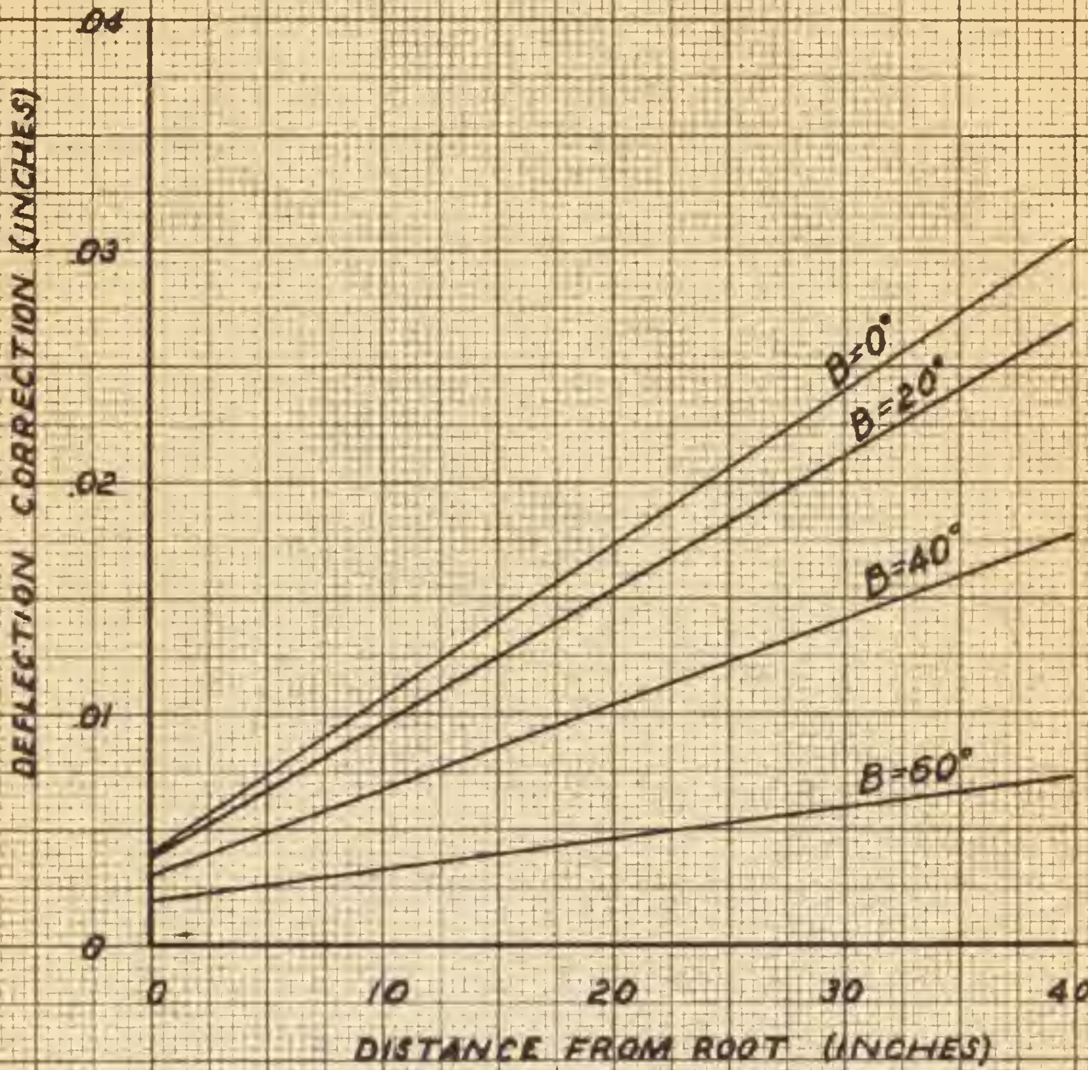


Figure 36

DEFLECTION CORRECTIONS DUE TO SAG OF SUPPORT FOR CONCENTRATED AND UNIFORM LOADS

Thesis
C37 Chandler
An investigation of
the stresses and deflec-
tions of swept plates.